A Matemática e Computação para Caracterização de Fluxo de Fluidos em Meios Porosos

Rafael March

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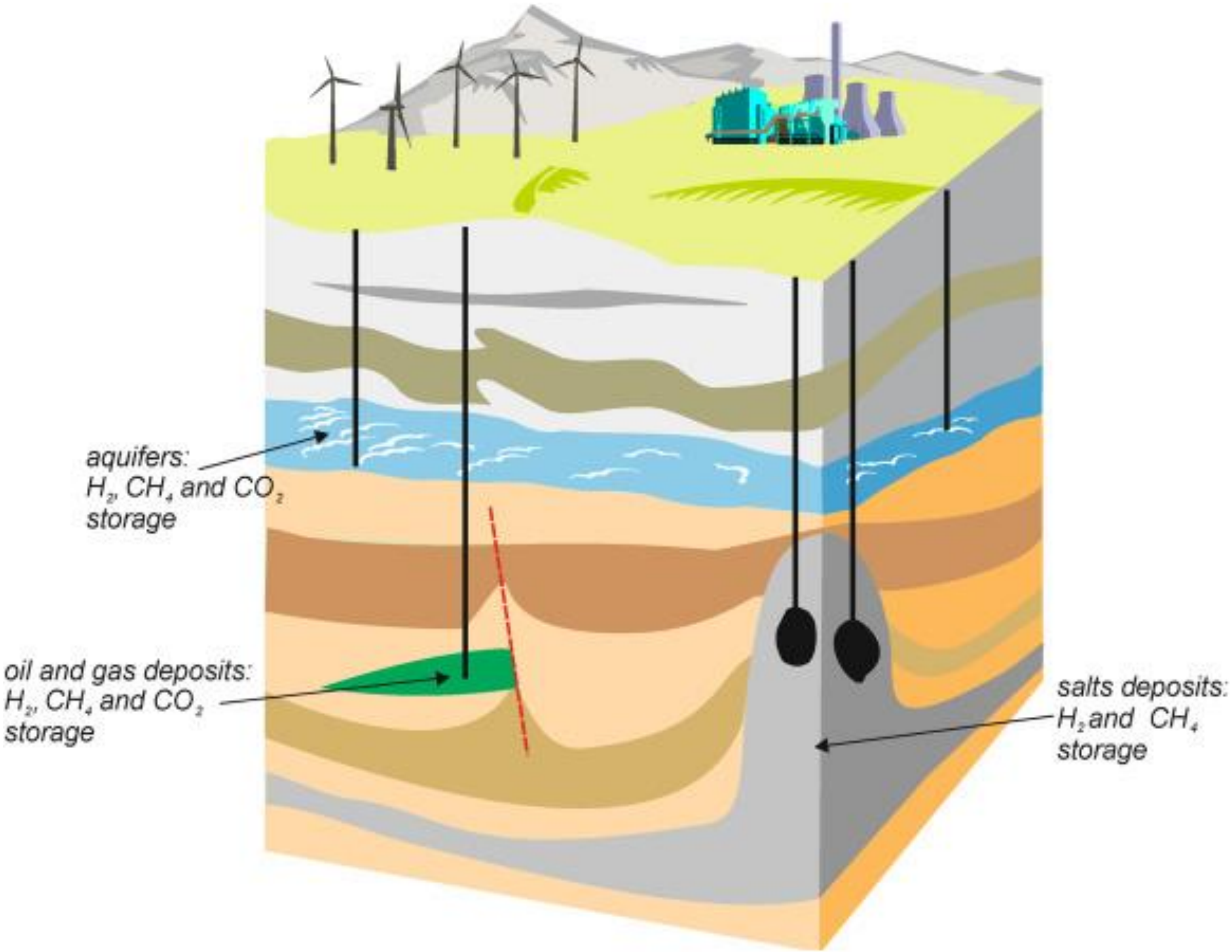
Porous Media Flow and GeoEnergy Applications



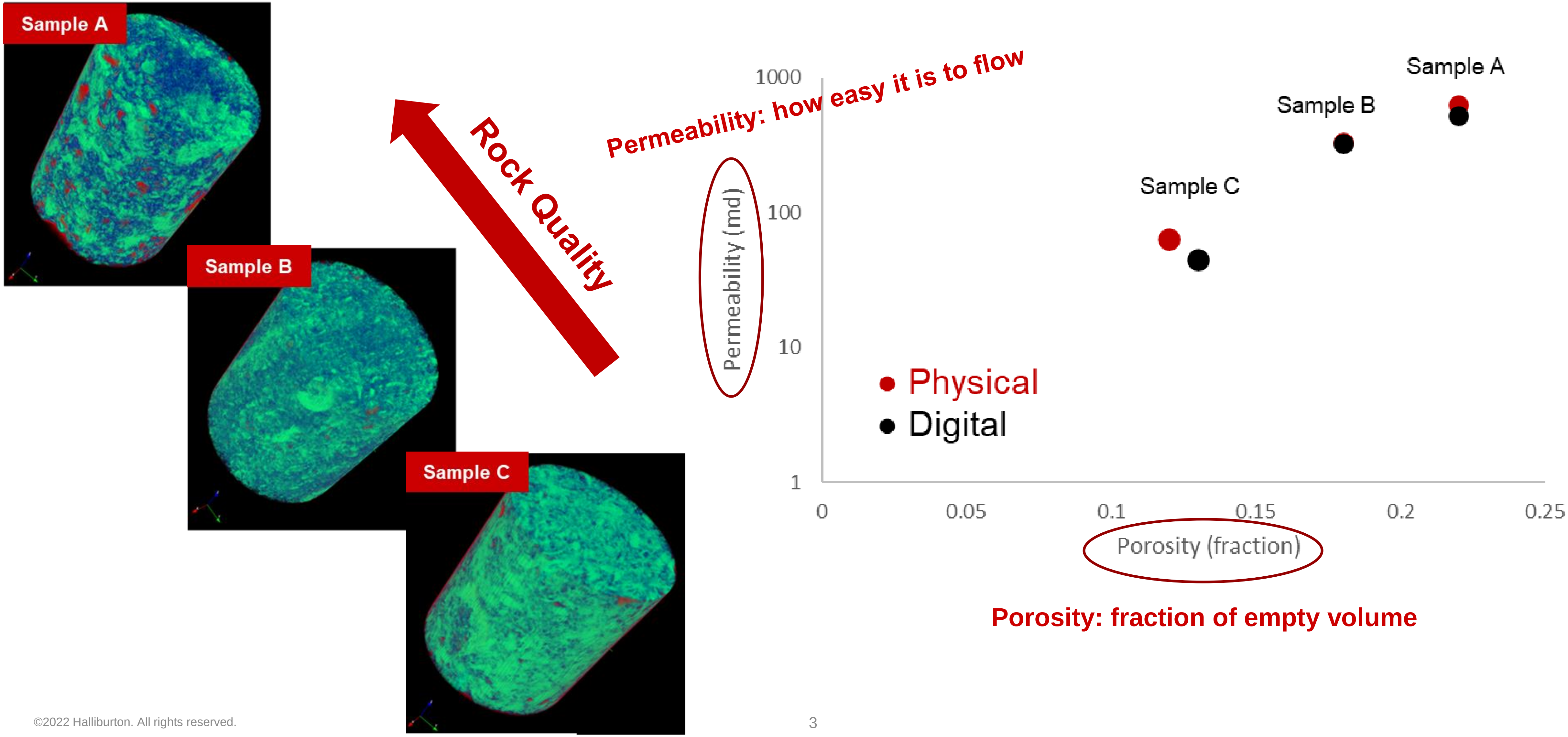
CO_2 Storage:
Preventing anthropogenic global warming

H_2 Storage
Enabling energy transition

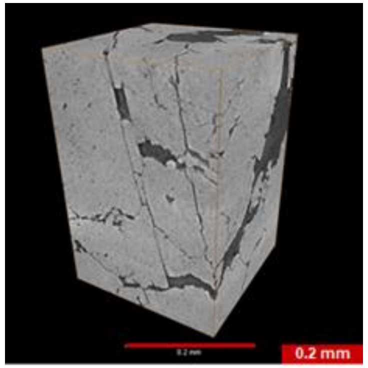
Oil and Gas Extraction
Ensuring we meet energy demand

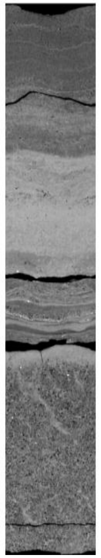
Figure taken from: https://doi.org/10.1016/j.ijhydene.2021.03.131

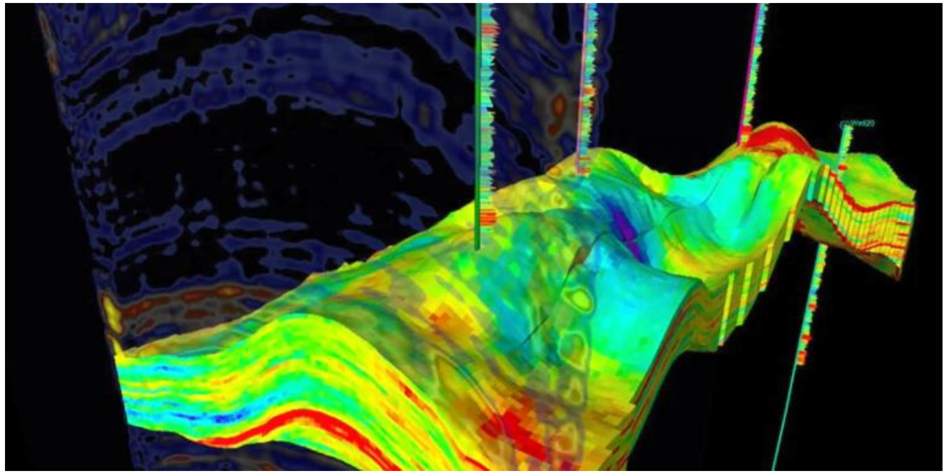
Porosity and Permeability: Two Important Concepts



Our Mathematical Models need to Adjust to the Scale of Interest







~mm **~cm** **~m** **~km**

NAVIER-STOKES EQUATIONS

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot \nabla \boldsymbol{u} = -\nabla p + \mu \nabla^2 \boldsymbol{u}$$

STOKES-BRINKMAN EQUATIONS

$$\nabla \cdot \boldsymbol{u} = 0$$

$$-\mu \nabla^2 \mathbf{u} + \boldsymbol{k}^{-1} \mathbf{u} + \nabla \mathrm{p} = 0$$

DARCY EQUATIONS

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u} = -\frac{\boldsymbol{k}}{\mu} \nabla p$$

PERMEABILITY: INPUT FROM SMALLER SCALES

Desafios na Caracterização de Fluxo de Fluidos em Fraturas

Outcrops across the globe

http://www.pbase.com/
http://blogs.egu.eu/
http://www.dgs.udel.edu/
http://archives.aapg.org/
https://www.uky.edu

Fractures and Faults are Everywhere!

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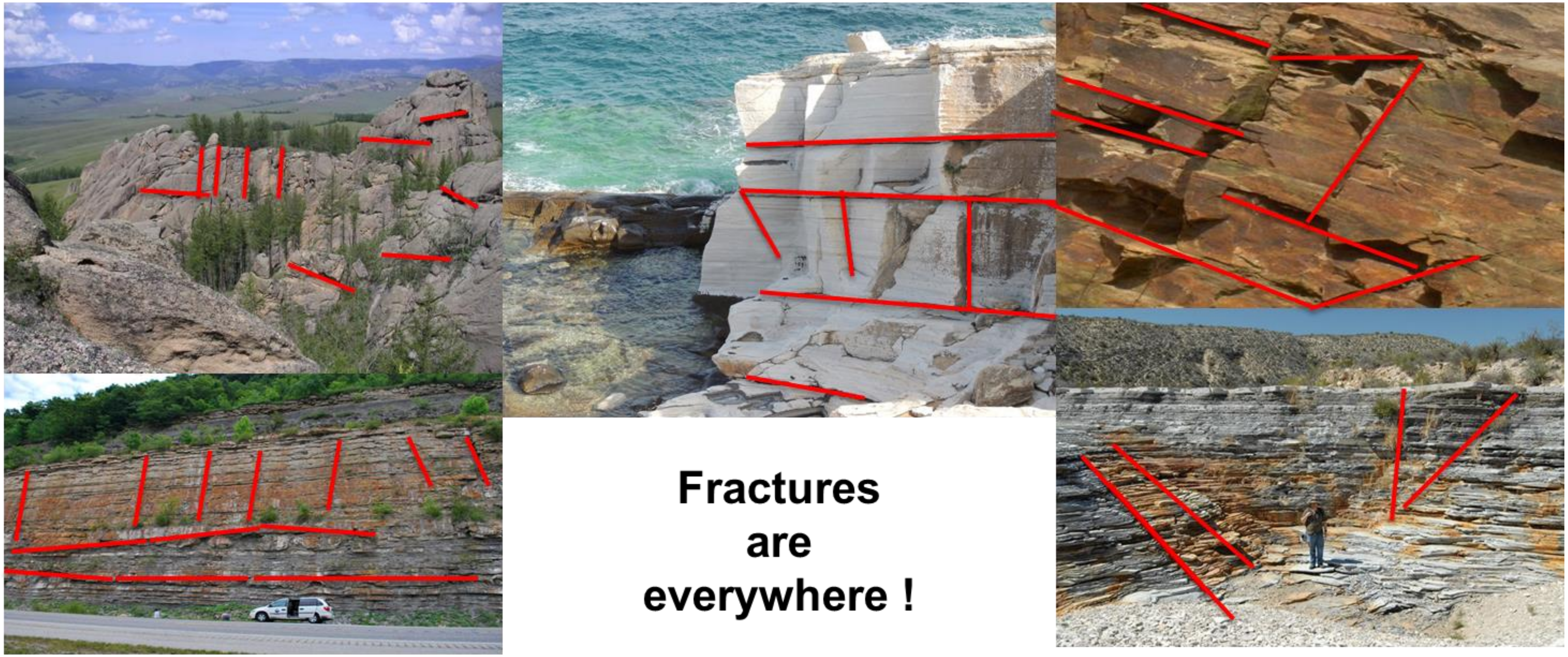
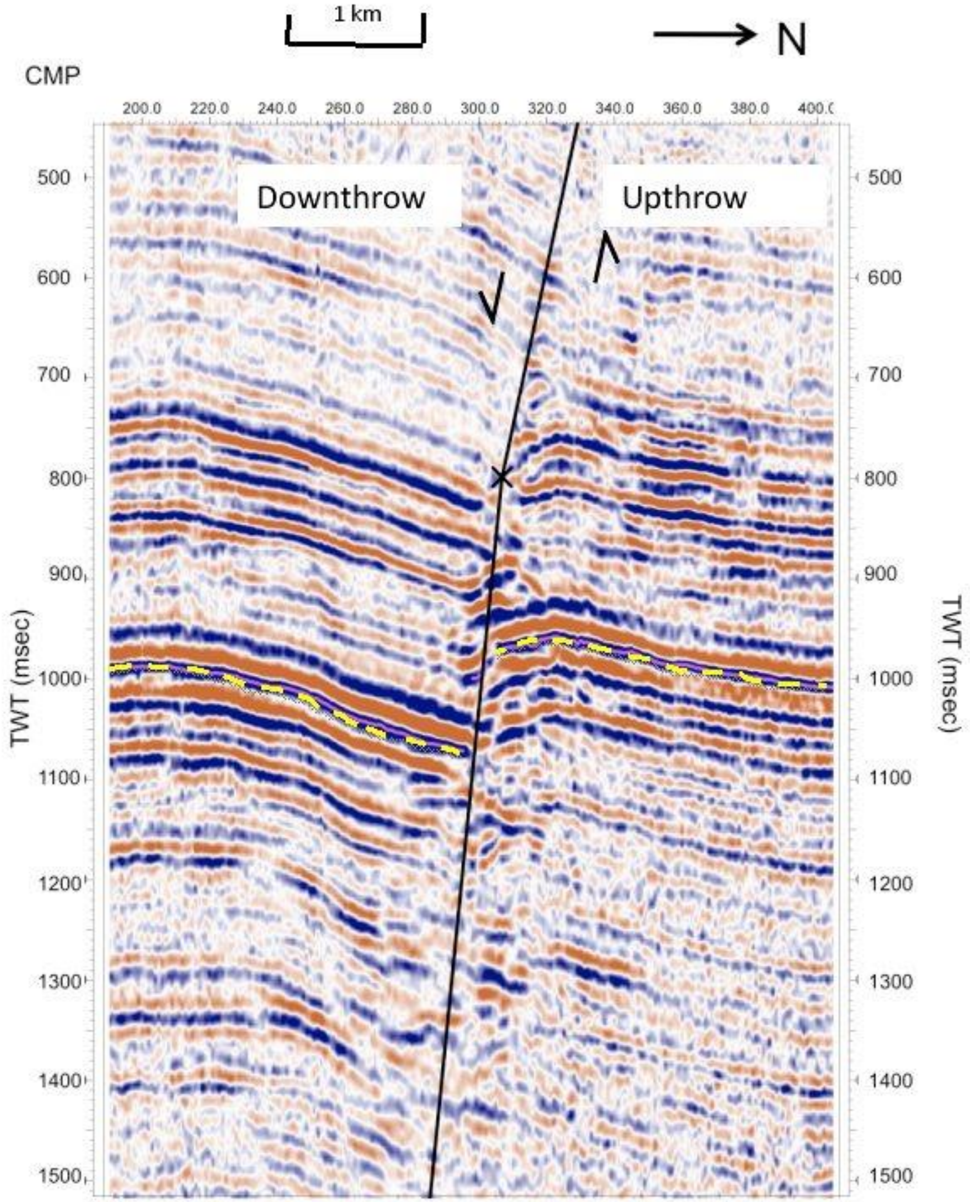


Cap Rock Leakage - Faults



 Fault figure taken from http://sub-surfrocks.co.uk/?page_id=260

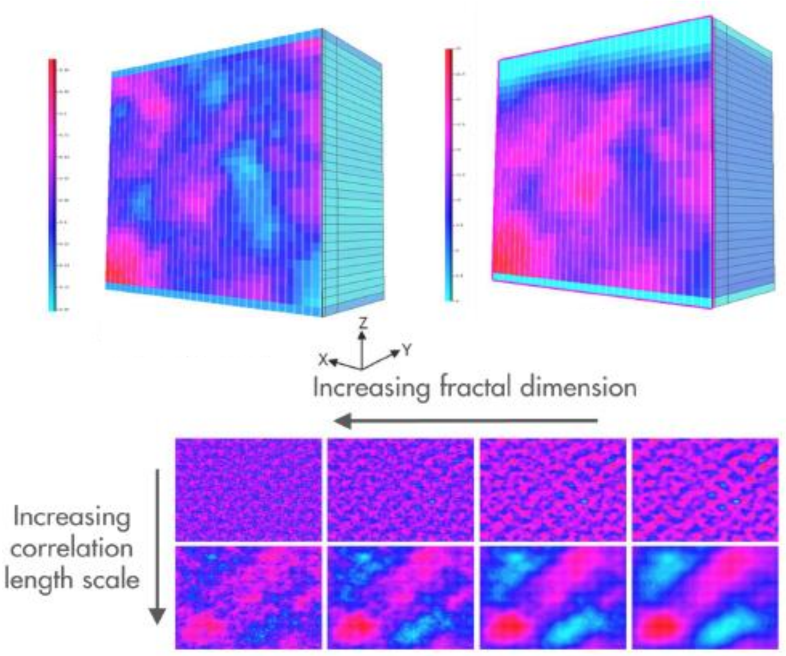
Cap Rock Leakage - Faults

Damage zones (fractures) associated to faults pose a serious leakage risk to underground gas storage

Fault outcrop, Crato Basin, Northeast Brazil

Multiscale Modelling is Needed

Fracture Plane Scale



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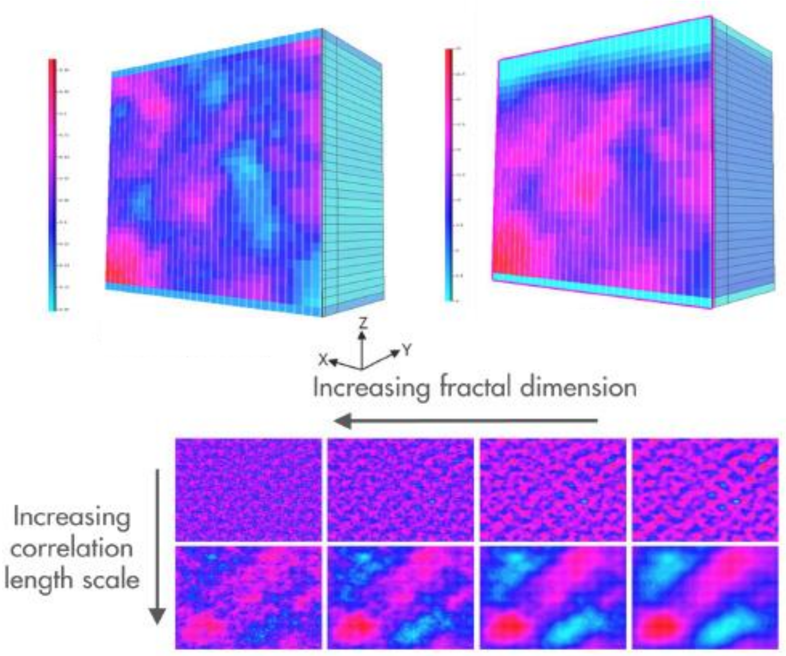
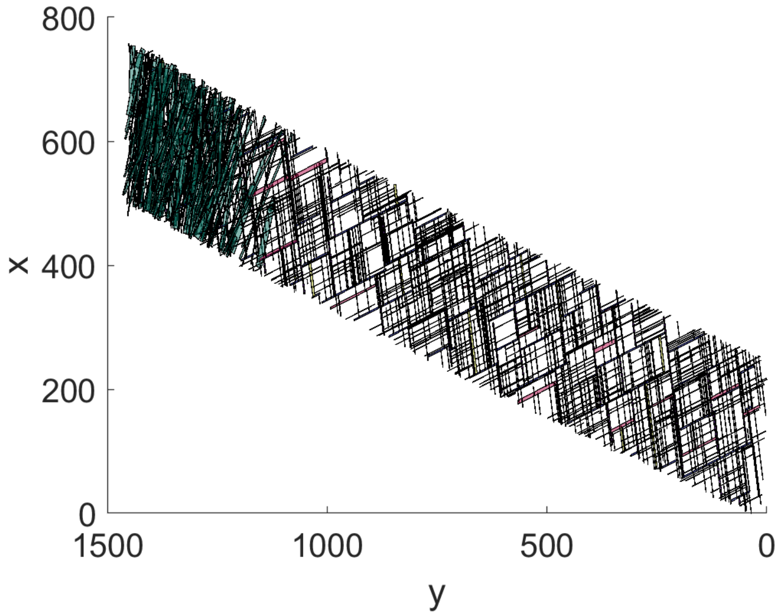
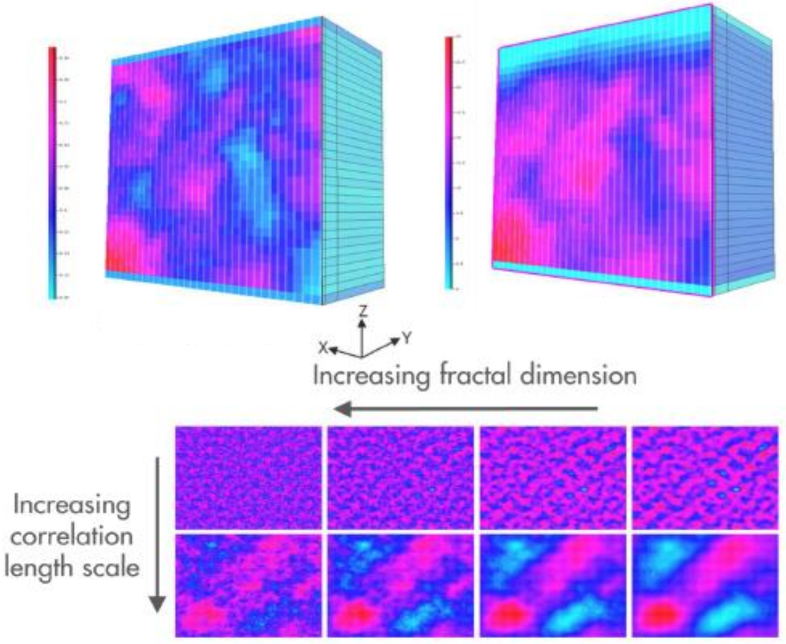




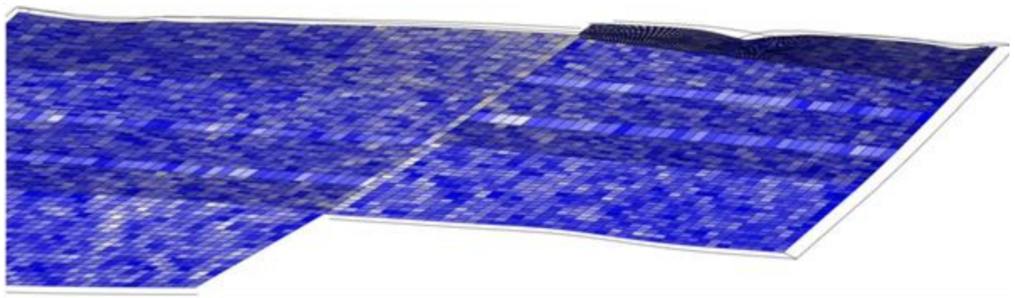
Fracture Network Scale

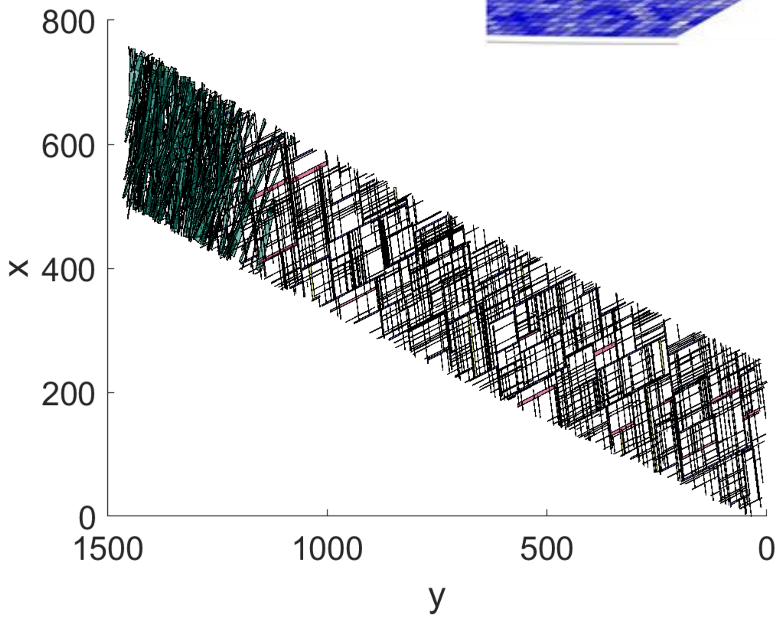
Multiscale Modelling is Needed

Fracture Plane Scale



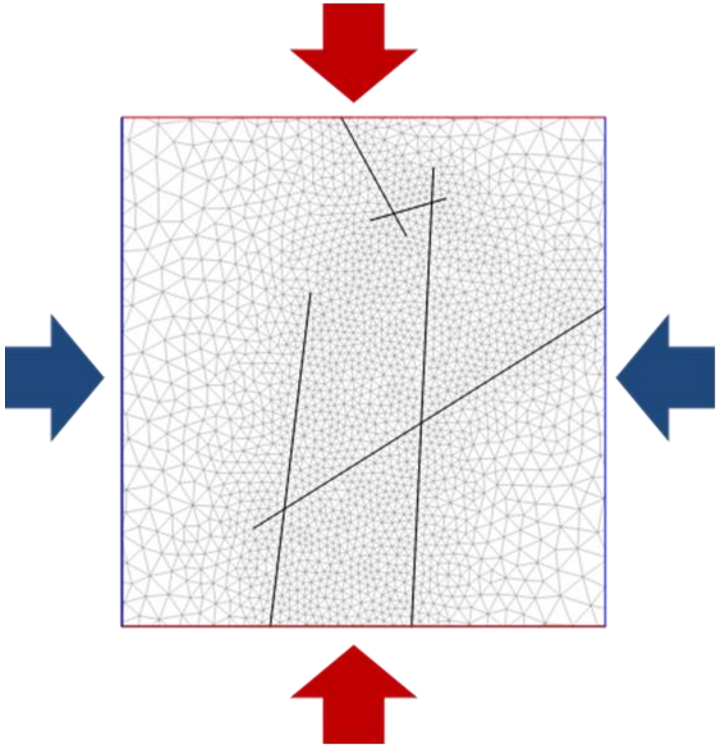
Basin/Reservoir Scale

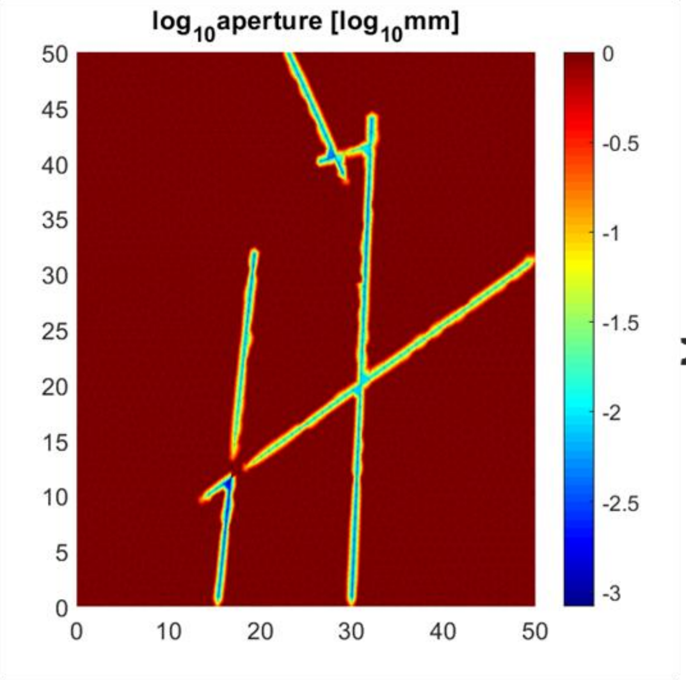


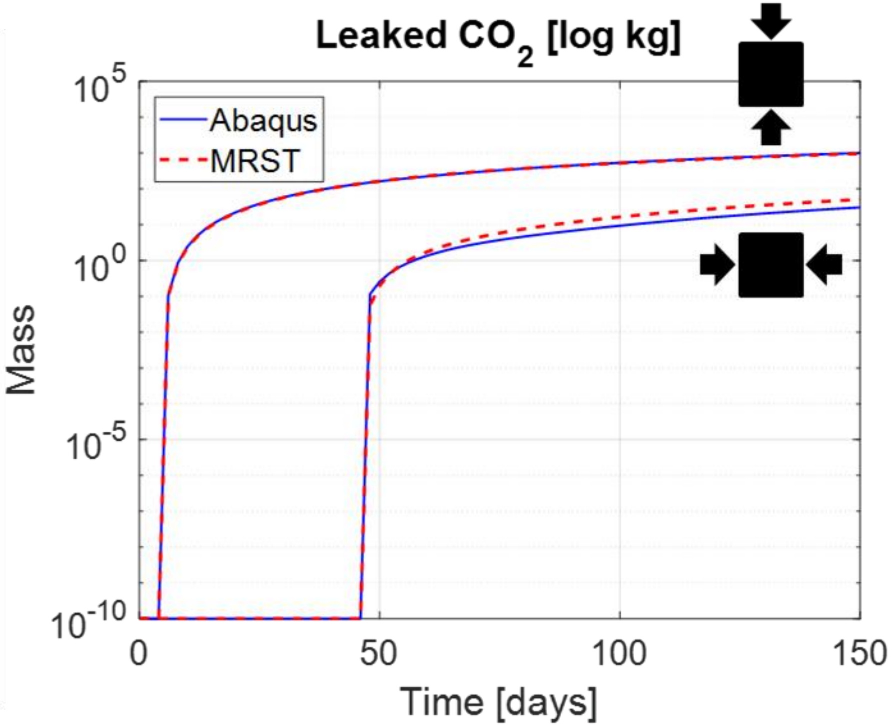


Fracture Network Scale

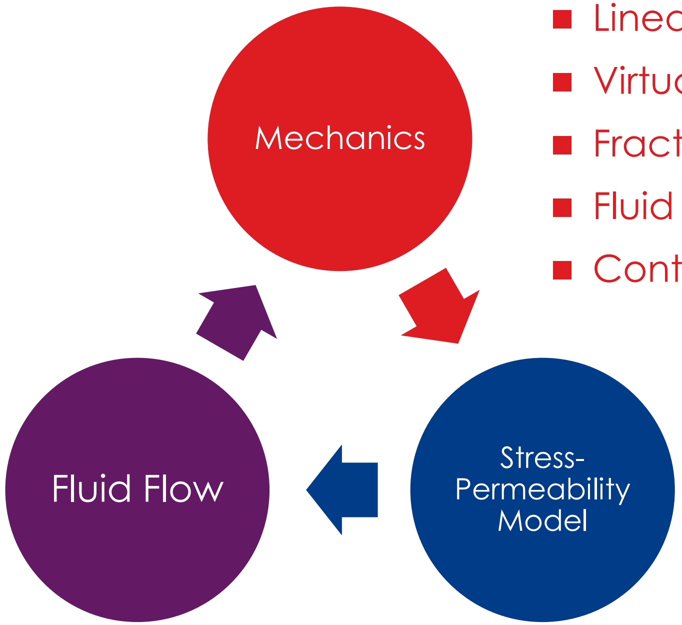
Fracture Network Permeability is Sensitive to In-Situ Stresses (Forces)







Workflow to Compute Stress-Sensitive Network Permeability



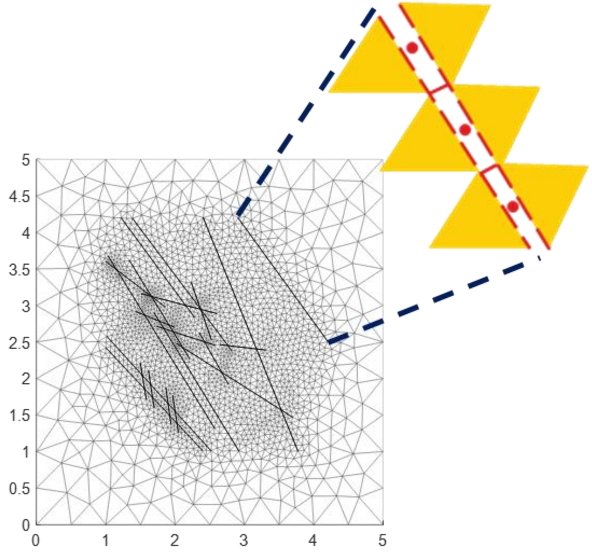
- Linear elasticity
- Virtual element method
- Fractures as discontinuities
- Fluid pressure as a traction
- Contact mechanics

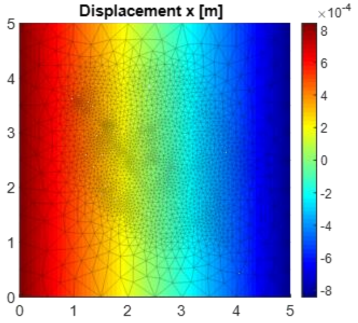
$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{f}$$

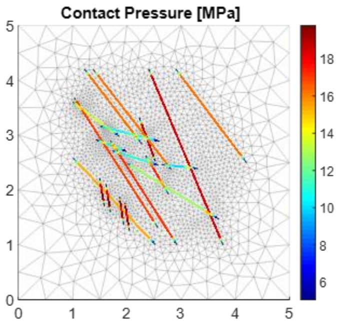
$$\boldsymbol{\sigma} = \boldsymbol{C} : \boldsymbol{\epsilon}$$

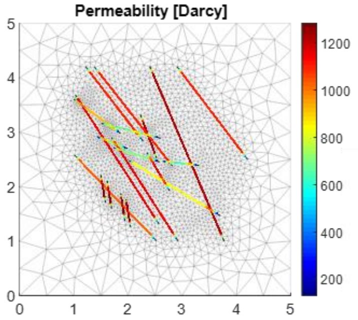
- Fracture-Matrix interaction
- Fluid pressure informs mechanics

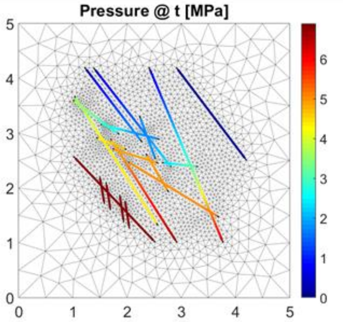
- Experiments
- Models (e.g. Barton & Bandis)





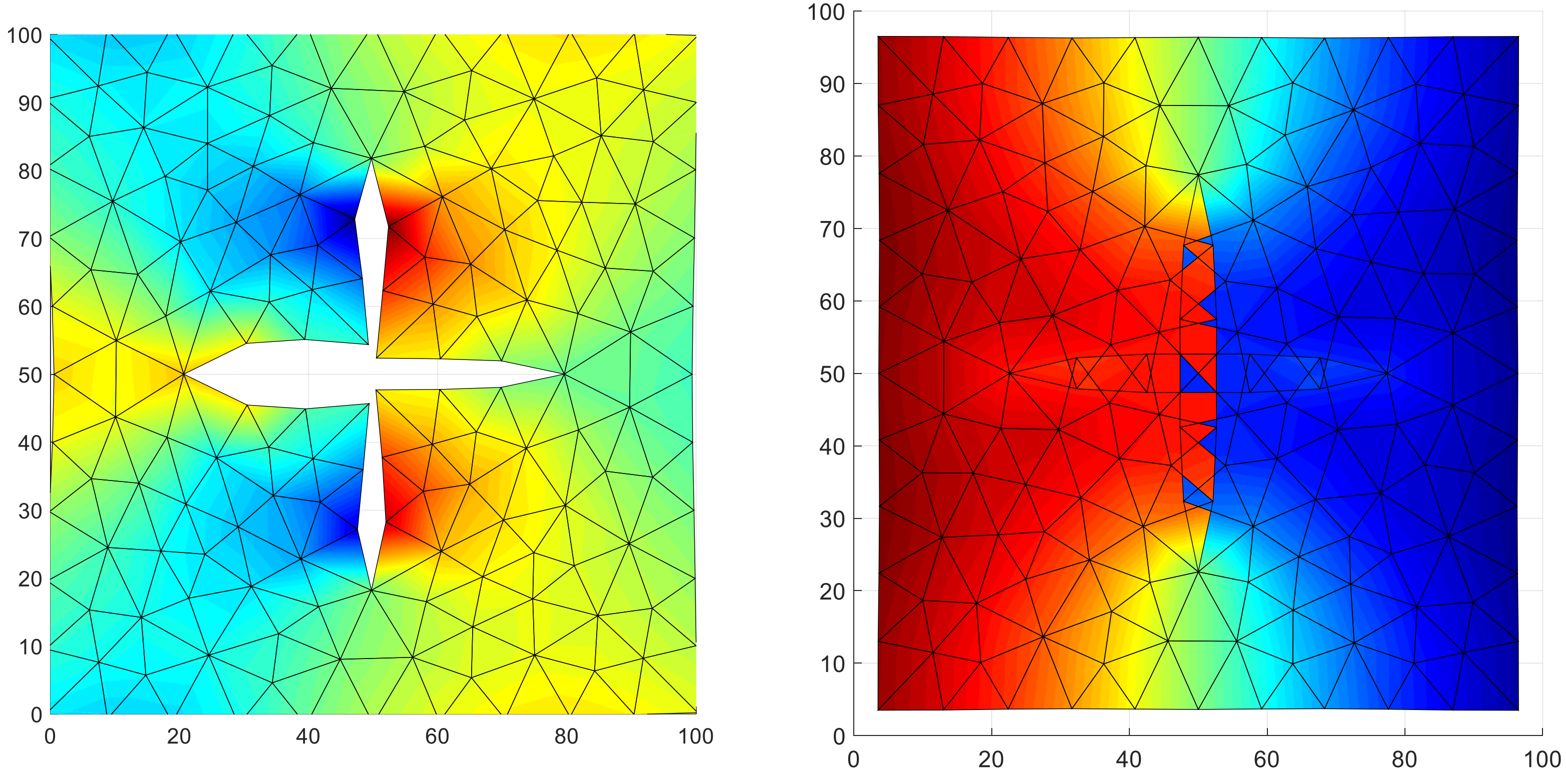






Fracture Meshing and Contact Mechanics

Fractures modelled as internal boundary conditions require proper treatment of contact mechanics in the fracture walls



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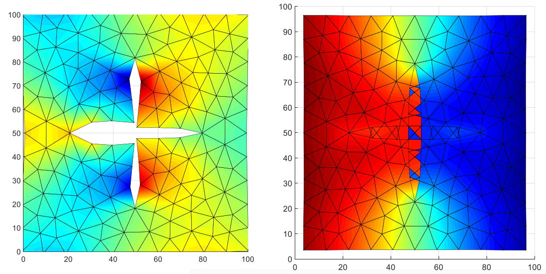
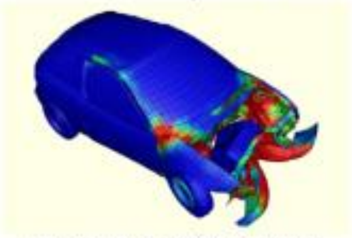
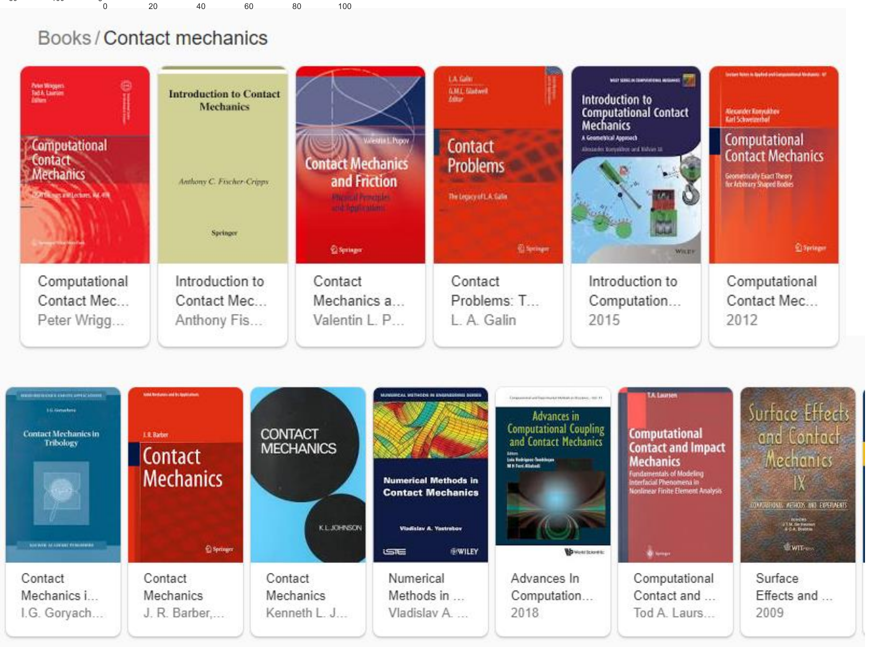


Crash-test www.porsche.com



[1] O. Klyavin, A. Michailov, A. Borovkov www.fea.ru



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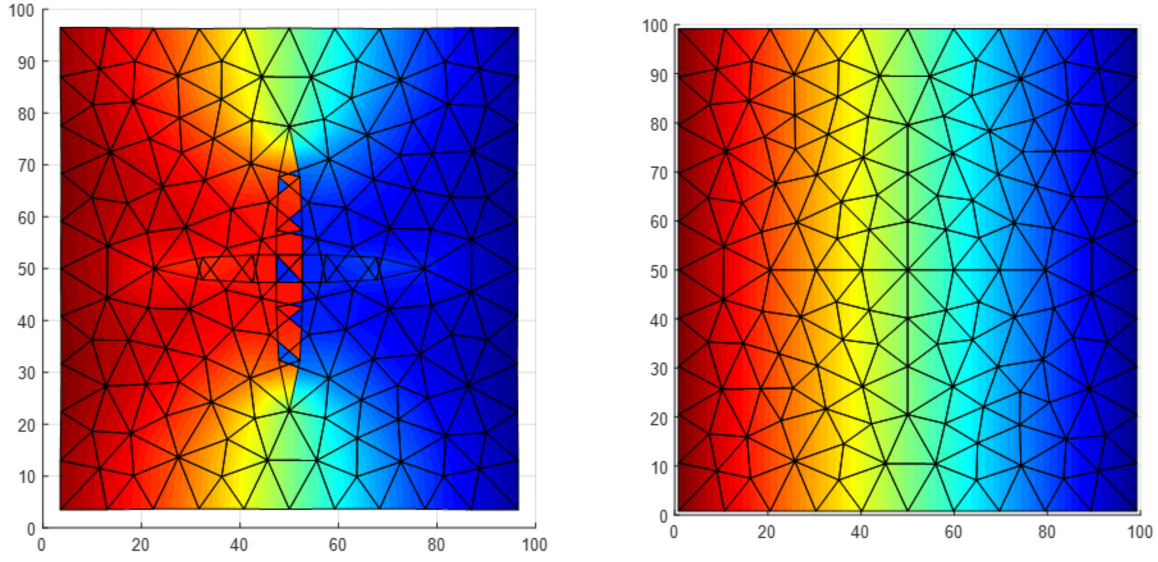
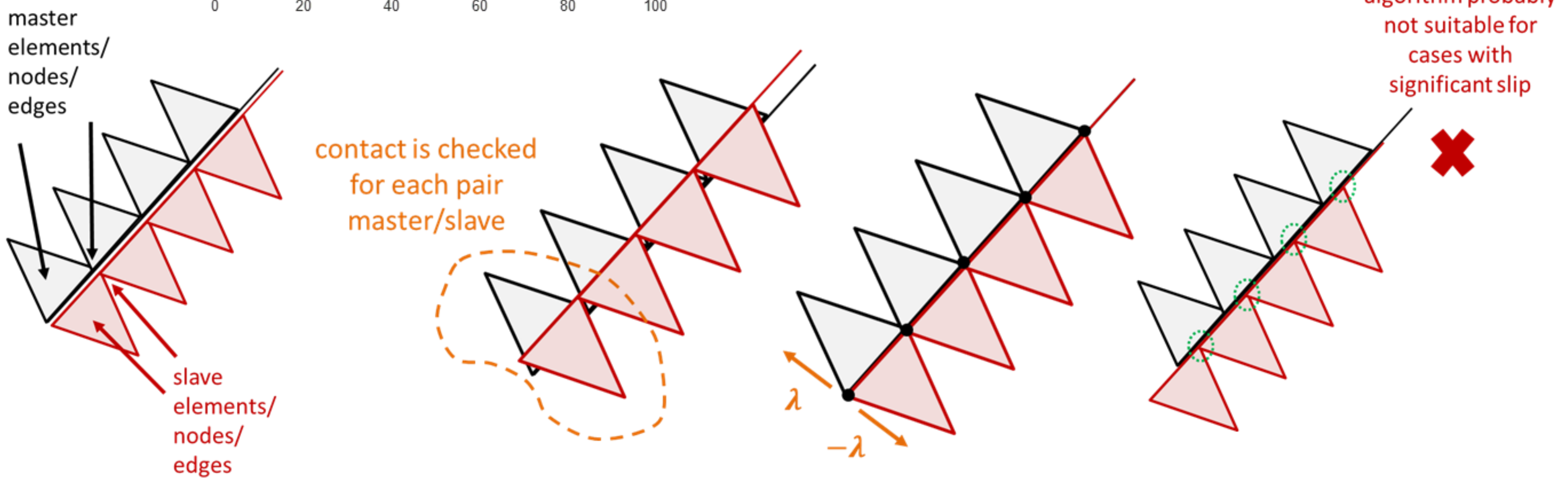
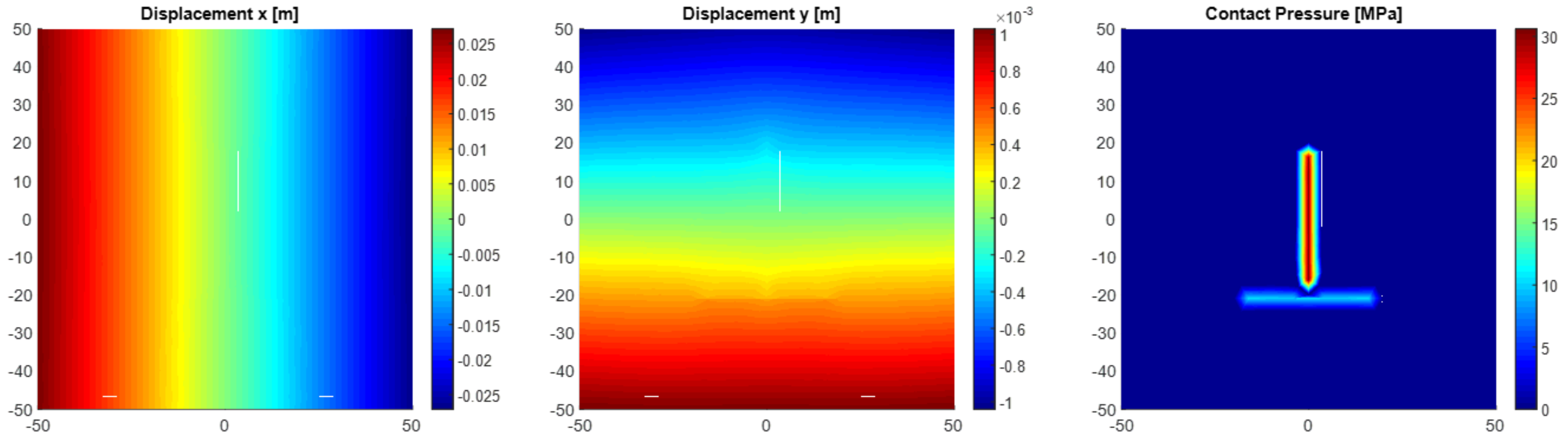


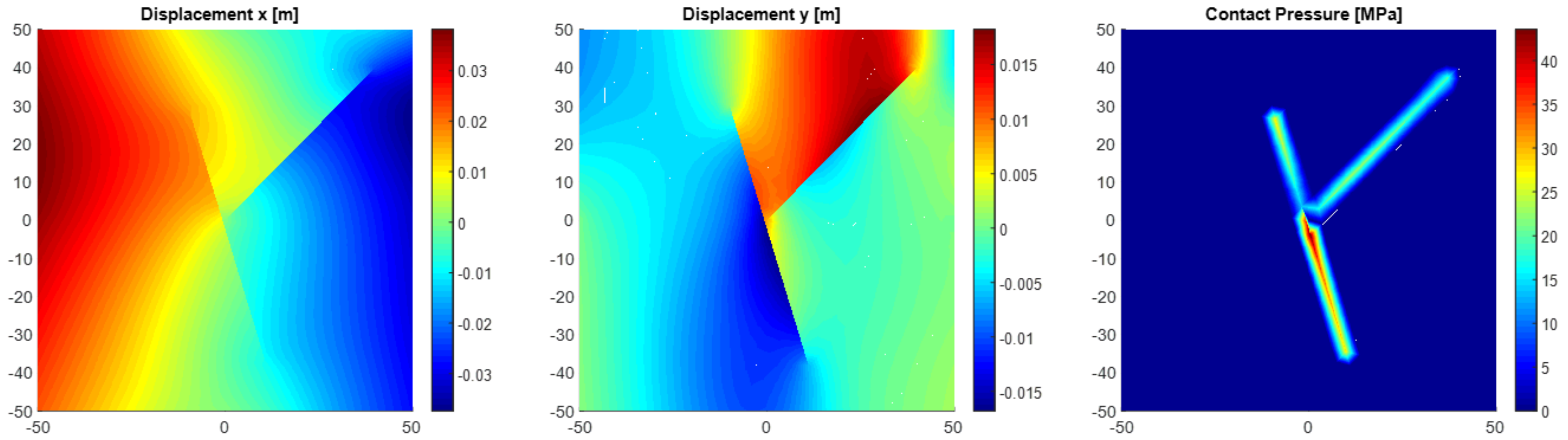


Comparison against Abaqus (Commercial Software)

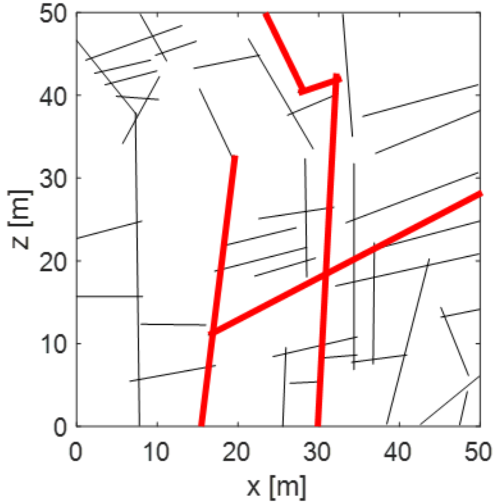
Experiment 1 (Abaqus results)



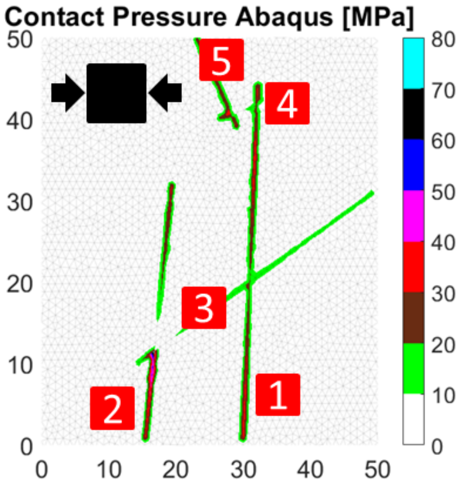
Experiment 2 (Abaqus results)

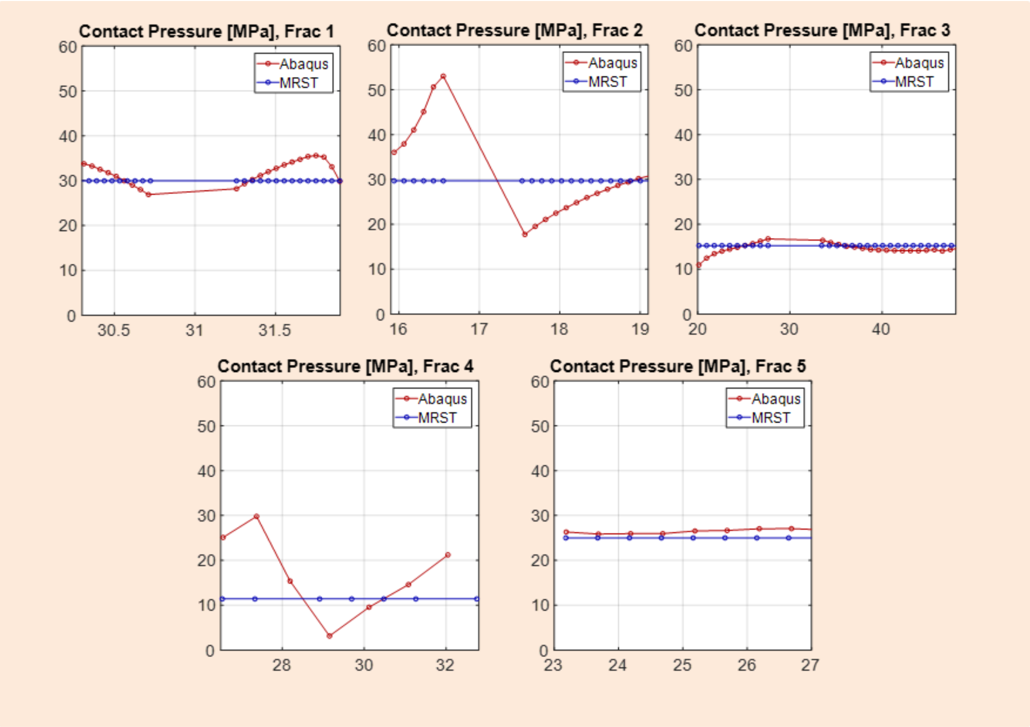


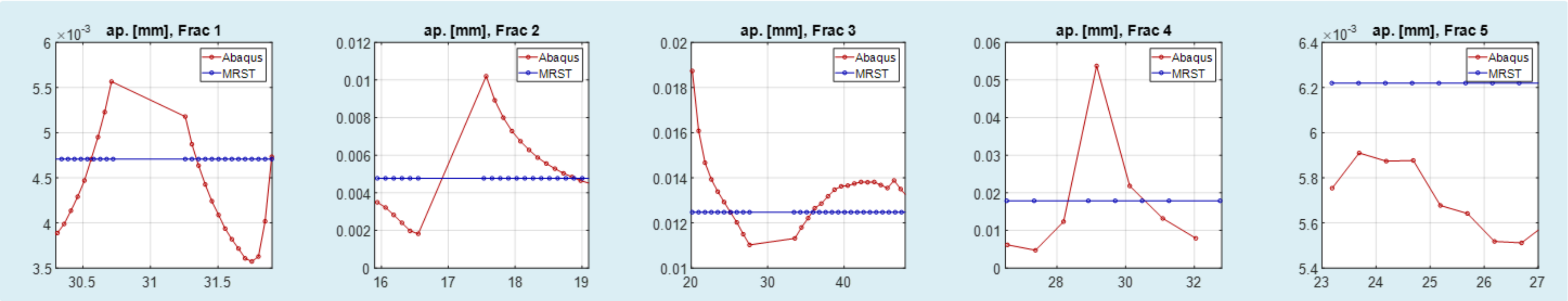
Comparison against Abaqus (Commercial Software)



Bisdom et al 2016







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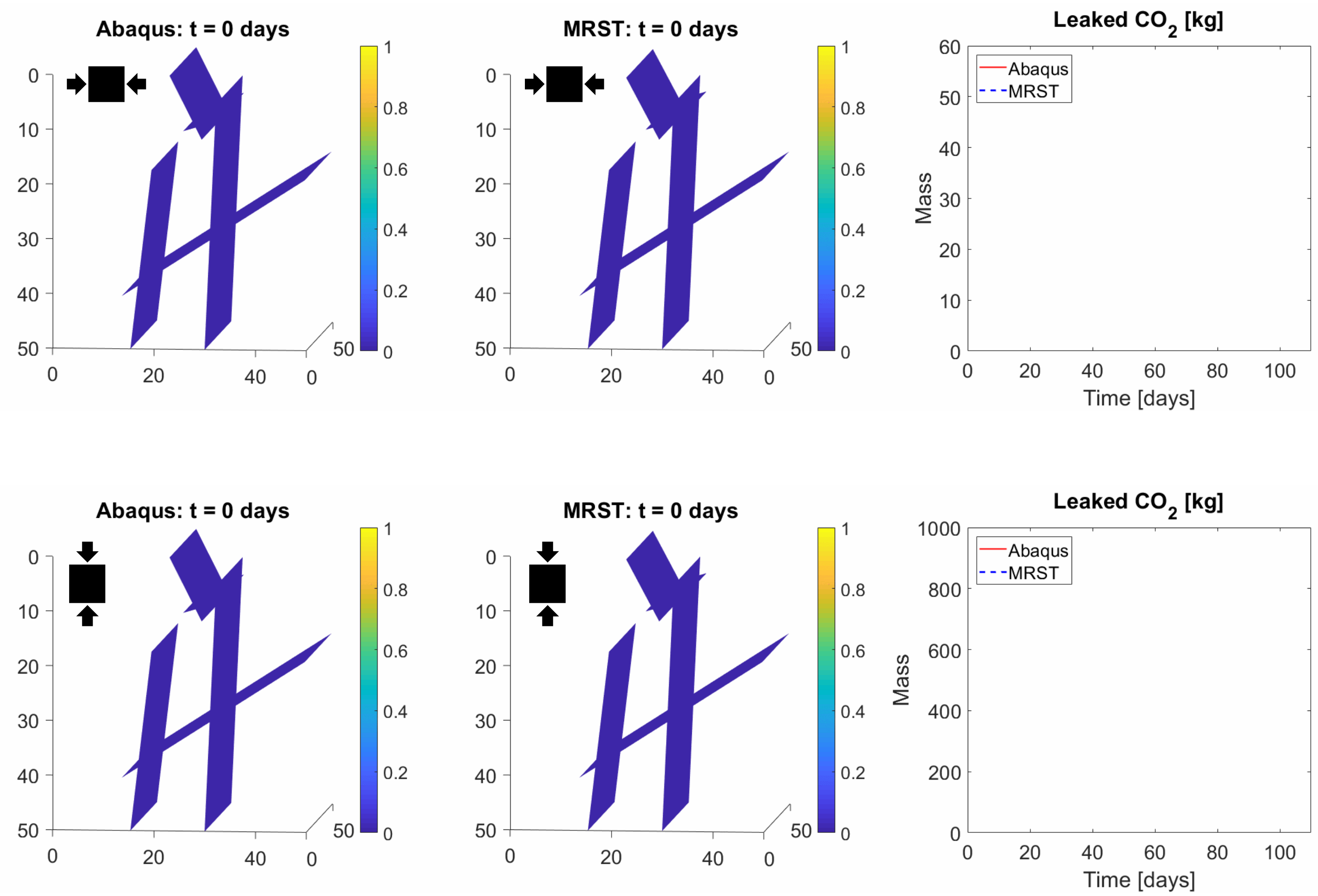


Application to Realistic 3D Fracture Network

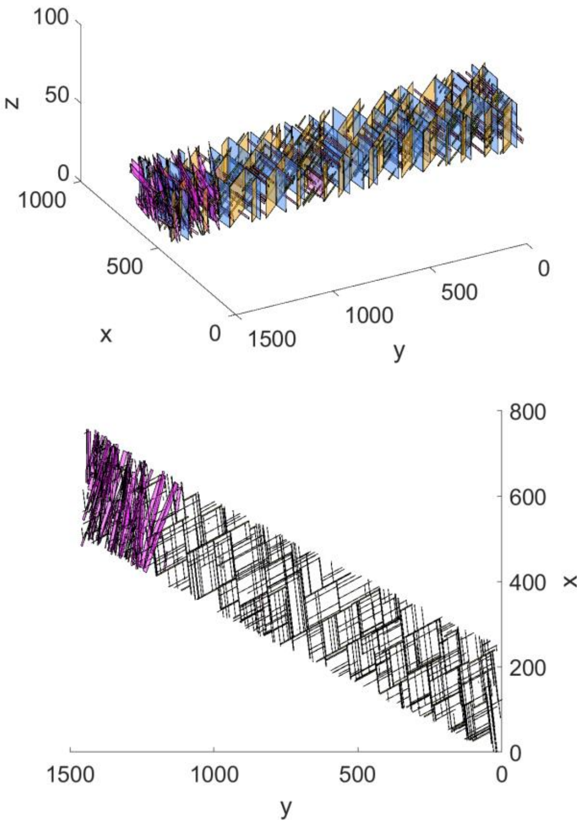
Goal: obtain *k* for the fracture network, given Δp

$$\nabla \cdot \mathbf{u} = 0$$

$$\boldsymbol{u} = -\frac{\boldsymbol{k}}{\boldsymbol{\mu}}\nabla \mathbf{p}$$

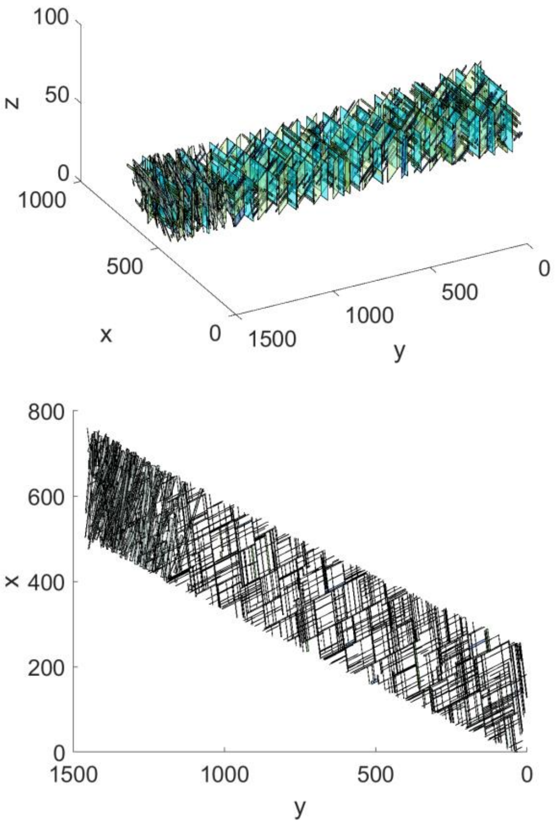
<u>Low density case</u>

~1346 fractures



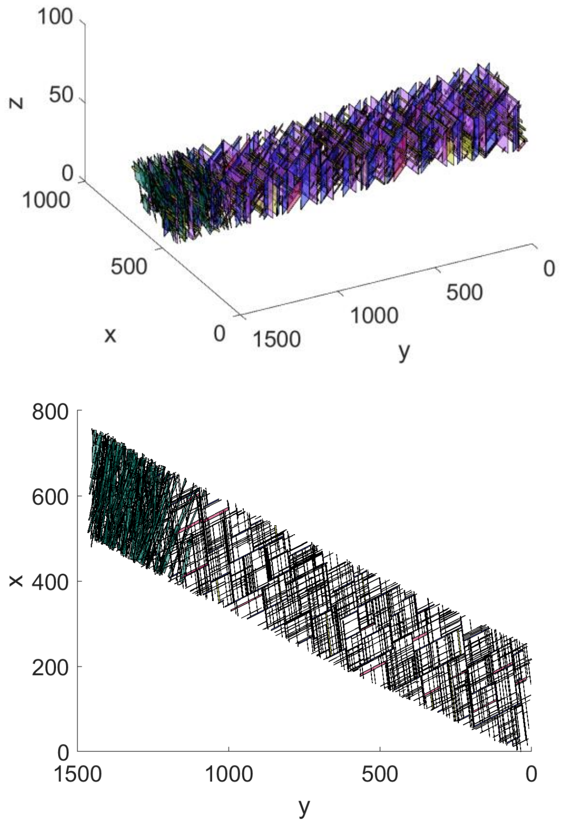
<u>Medium density case</u>

~2247 fractures



<u>High density case</u>

~3117 fractures



Fracture Plane Stress-Permeability Model

$$k_{core} = k_m + \left(\frac{1}{12}\right)\left(\frac{4}{\pi d}\right) b^3$$

$$b = b_0 - \Delta b$$

$$\Delta b = R \cdot b_0 (1 - e^{-\alpha \sigma_n^{\beta}})$$

$$k_m = 10^{-20}\ m^2$$

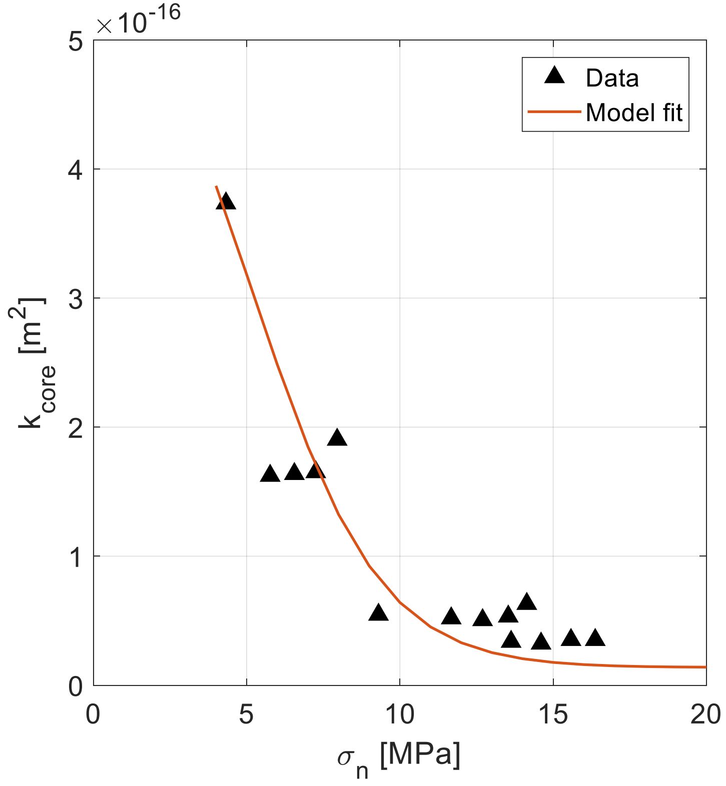
$$d = 0.2574\ m$$

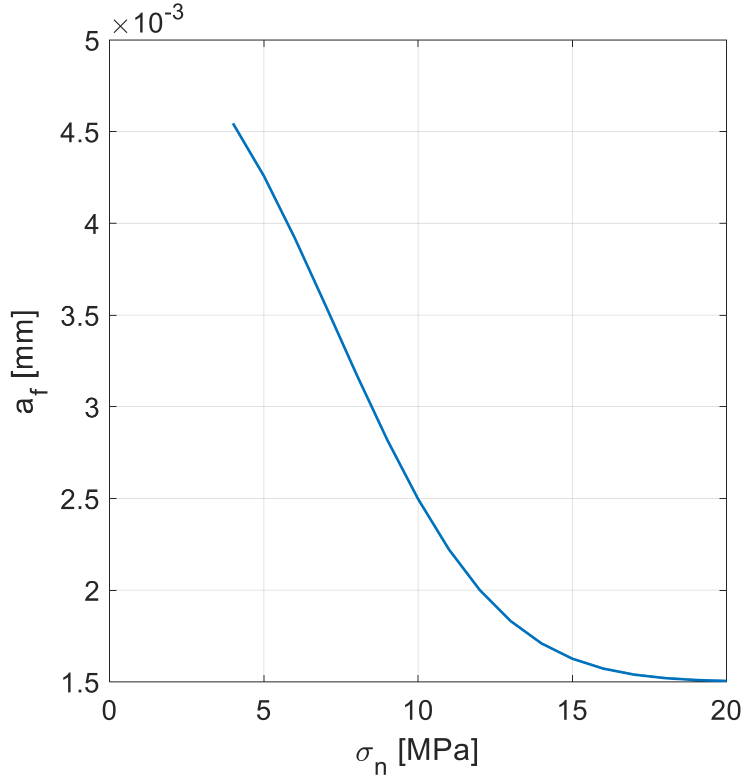
$$\beta = 2.4$$

$$\alpha = 0.005$$

$$R = 0.7$$

$$b_0 = 0.005\ mm$$

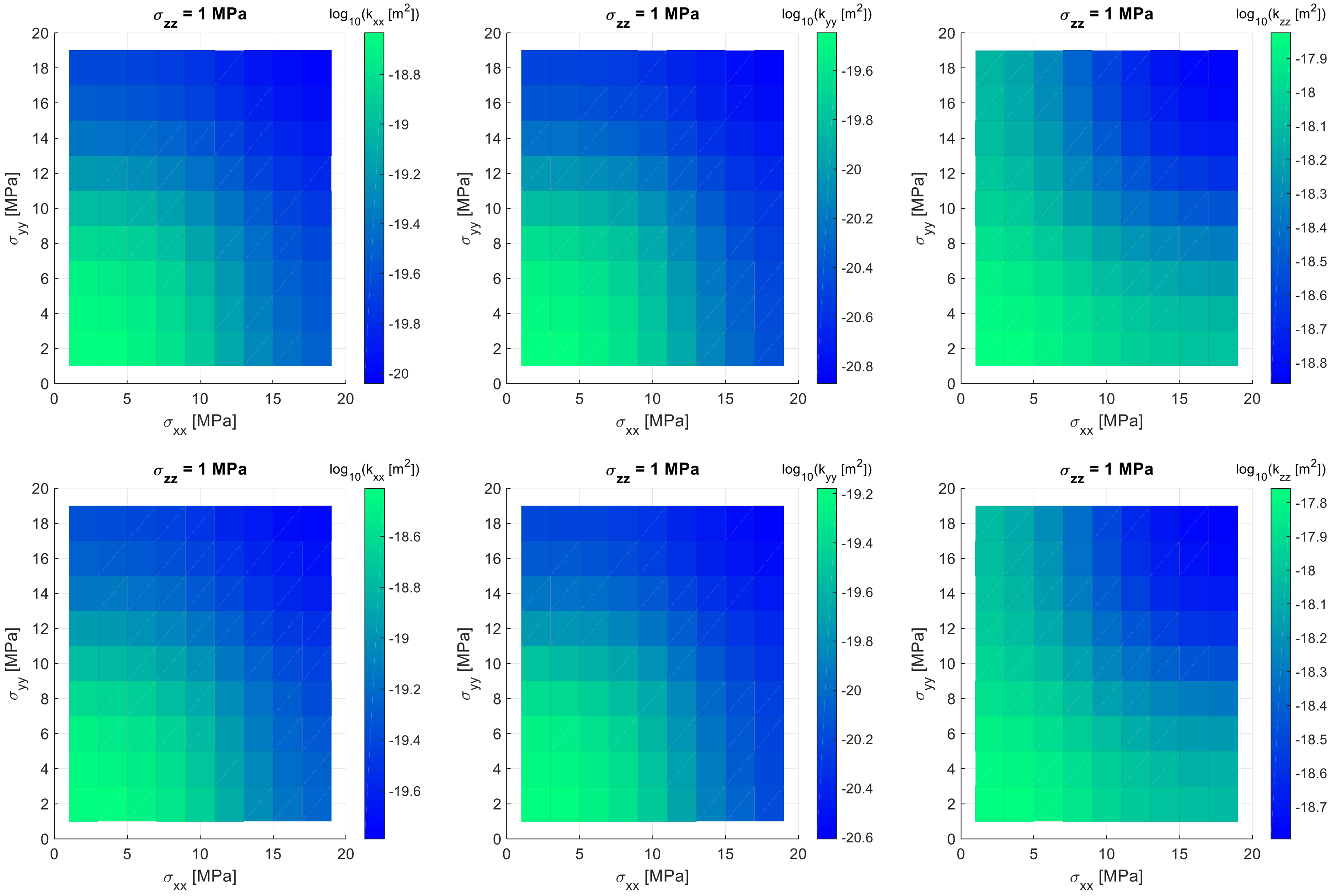




Results for $\sigma_{zz} = 1\ \mathrm{MPa}$

Low density case

High density case

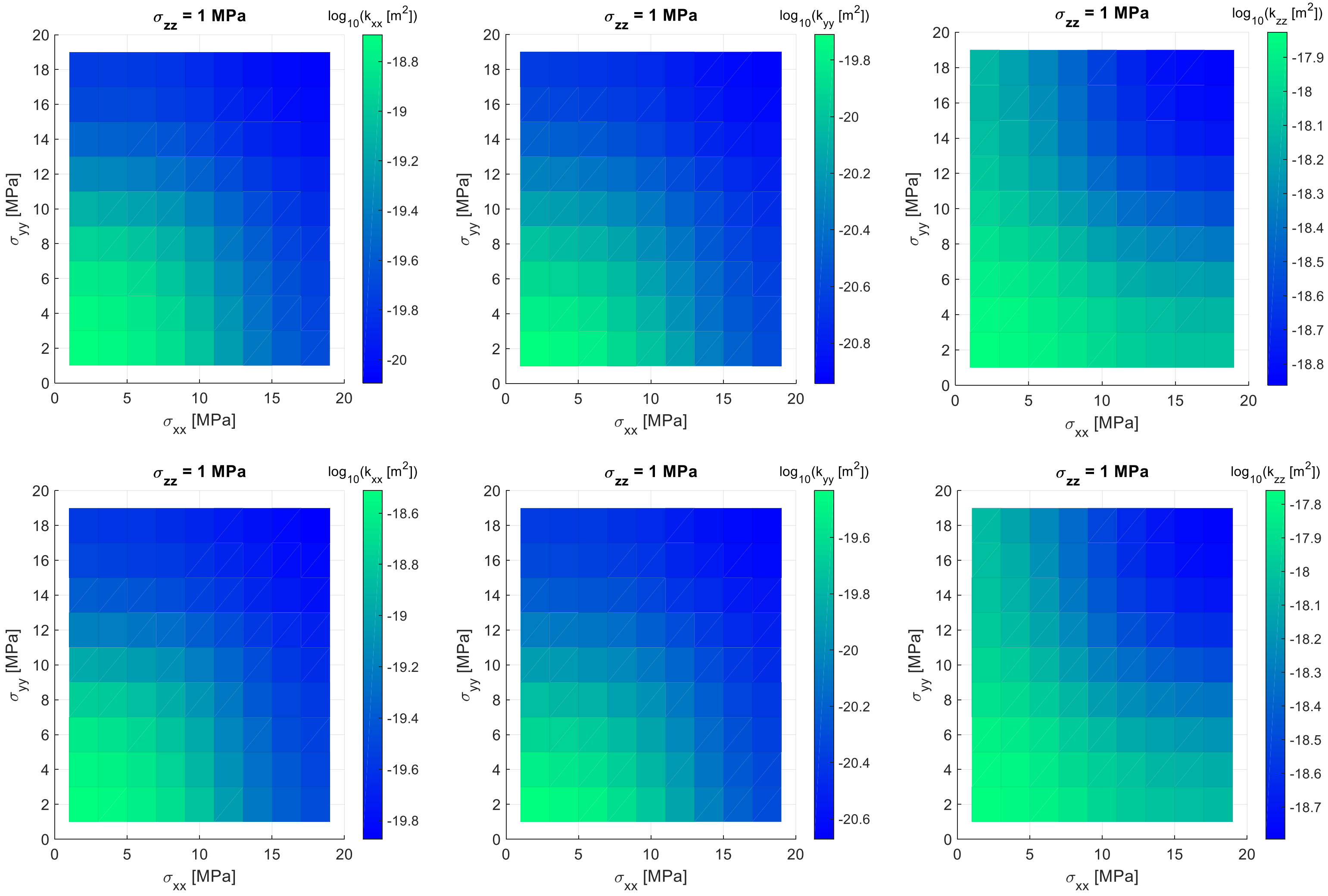


- Permeability changes with increasing stress: a bit over one order of magnitude in all three directions ($0 < \sigma_{xx}, \sigma_{yy}, \sigma_{zz} < 20$ MPa)
- Considering a reference matrix permeability of ~10^{-20} m^2, permeability increase caused by fractures is between 1 and 2 orders of magnitude
- $k_{zz} > k_{xx} > k_{yy}$
- No significant changes with increasing σ_{zz}
- High density case shows little improvement in permeability in comparison to low density case

Results for $\sigma_{zz} = 19\ \mathrm{MPa}$

Low density case

High density case



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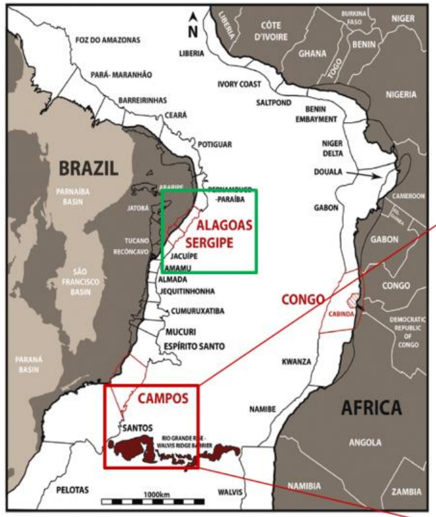
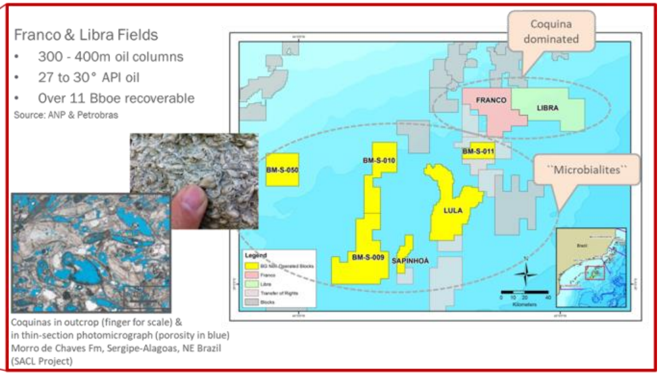
Física de Rocha Digital

Como a matemática e tecnologias digitais ajudam no entendimento e caracterização das rochas

Digital Rock Physics as a Tool to Characterize a Reservoir

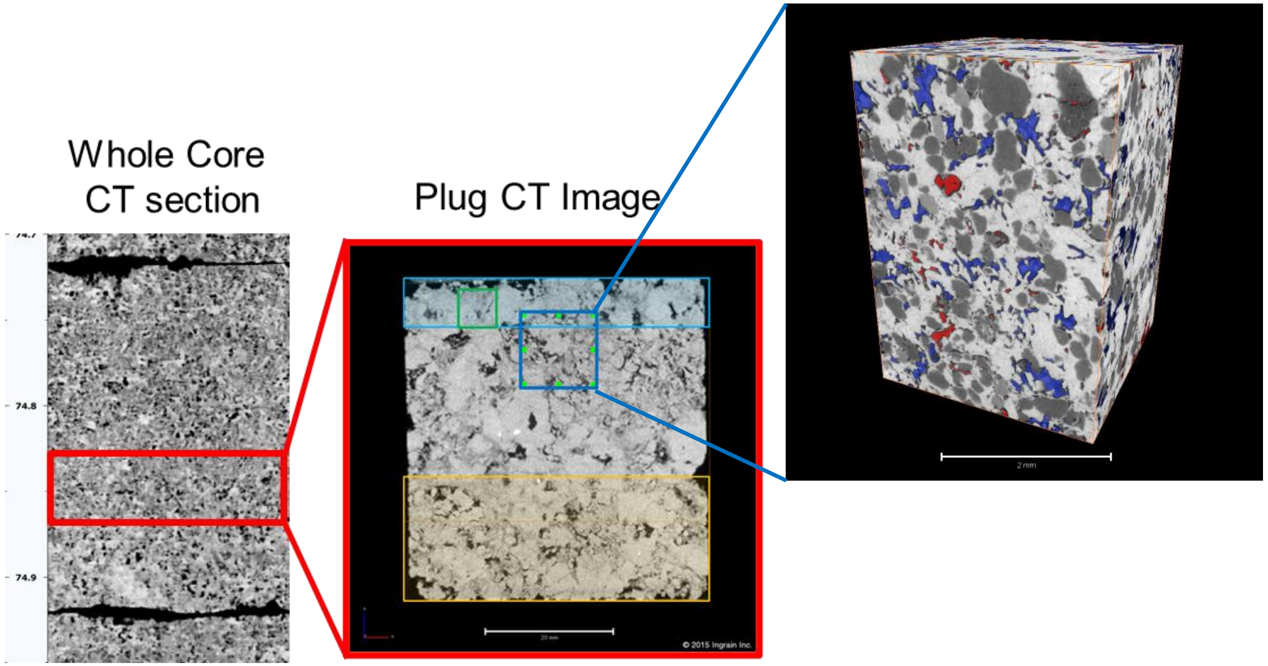


Guardado et al., 1989 and Carvalho et al., 2000; Thompson et al., 2015



Whole Core CT section

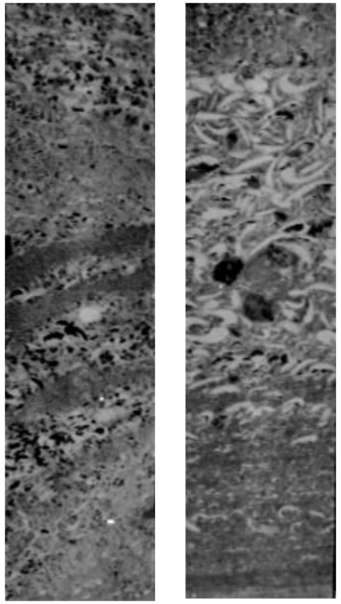
Plug CT Image



Digital Computation of Permeability Tensors

- Permeability is an anisotropic property, and it is mathematically described by a symmetric second order tensor
- Anisotropic permeability behavior exists at all scales
- Typical upscaling workflows consider only the principal components of the permeability tensor
- Here we analyze the full-tensor behavior of small carbonate rock samples and suggest a workflow to upscale full-tensor permeability to larger samples

Cores



Darcy's Law

$$\boldsymbol{u} = -\frac{\boldsymbol{k}}{\mu}\nabla p$$

$$\boldsymbol{k} = \begin{pmatrix} \boldsymbol{k_{xx}} & k_{xy} & k_{xz} \\ sym. & \boldsymbol{k_{yy}} & k_{yz} \\ sym. & sym. & \boldsymbol{k_{zz}} \end{pmatrix}$$

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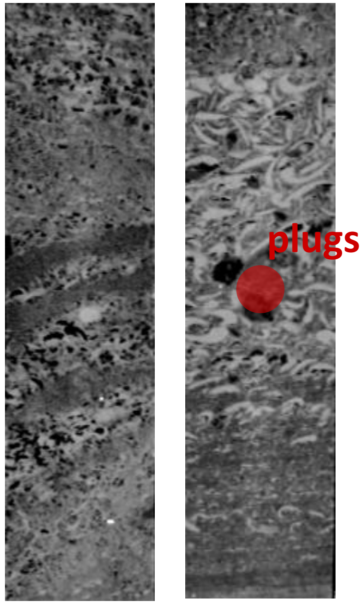
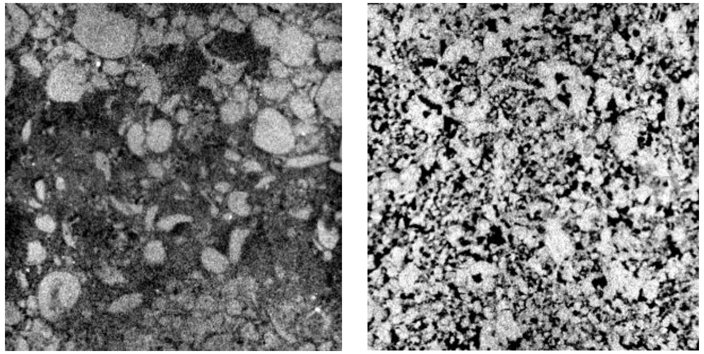


Plugs



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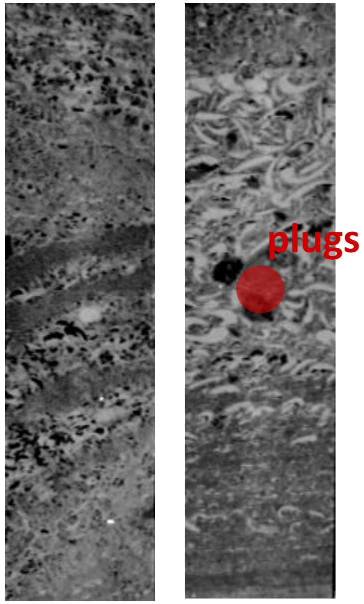

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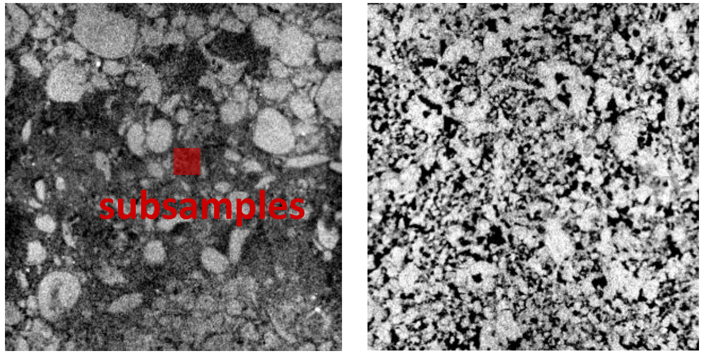
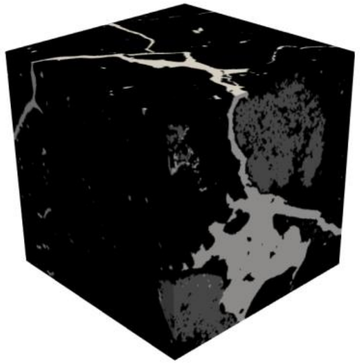


Subsamples



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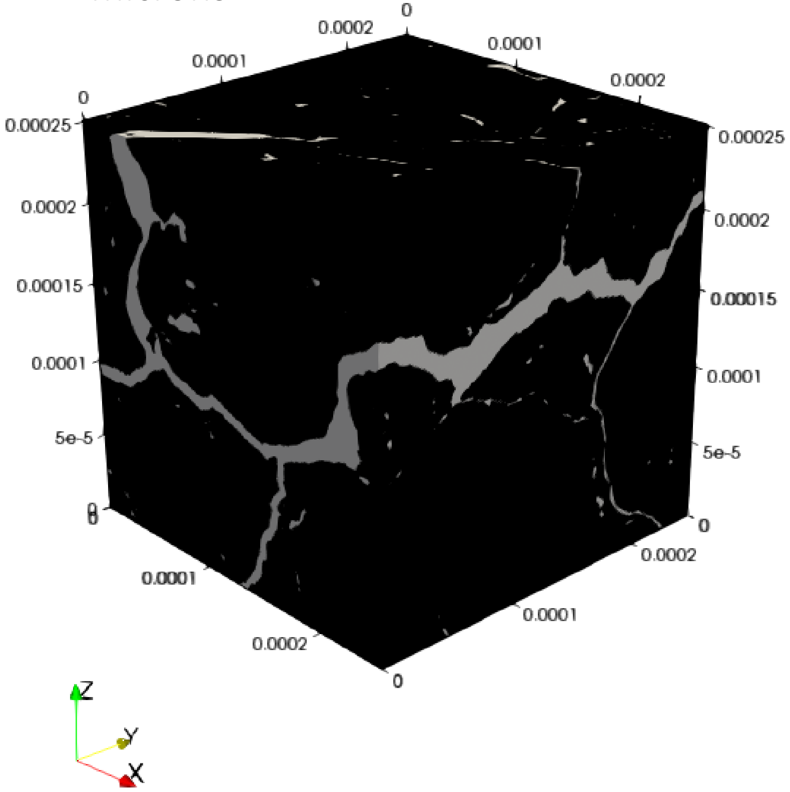
$$\boldsymbol{k} = \begin{pmatrix} \boldsymbol{k_{xx}} & k_{xy} & k_{xz} \\ sym. & \boldsymbol{k_{yy}} & k_{yz} \\ sym. & sym. & \boldsymbol{k_{zz}} \end{pmatrix}$$

Subsamples under Analysis

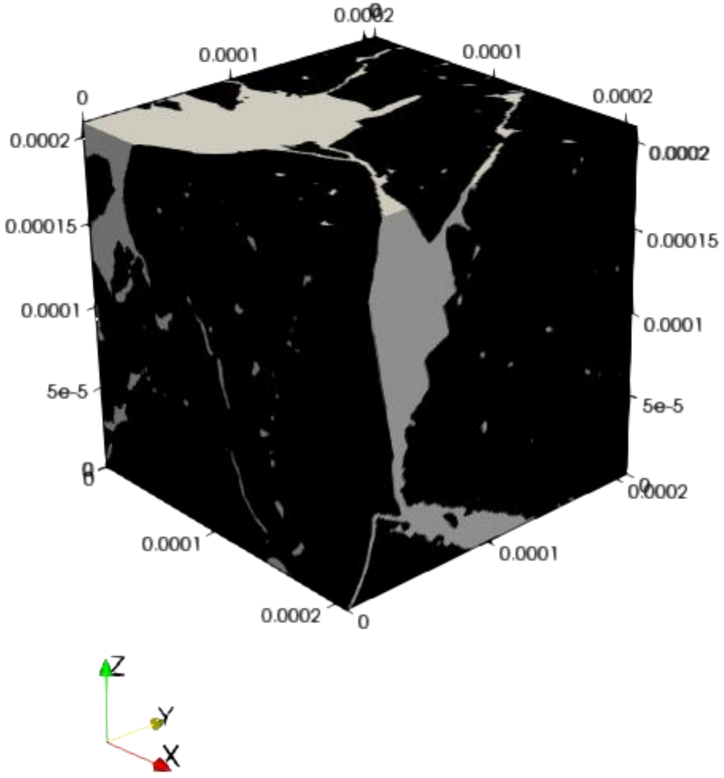
Sample A

- nx = 210, ny = 210, nz = 210
- Lx = 0.25 mm, Lx = 0.25 mm, Lz = 0.25 mm
- dx = 1.2 microns, dy = 1.2 microns, dz = 1.2 microns

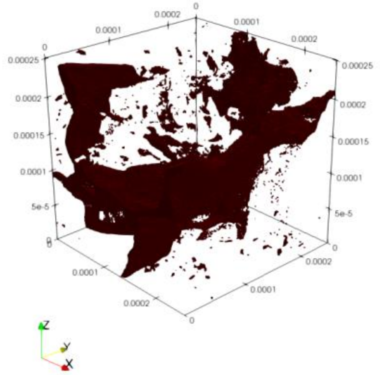


Sample B

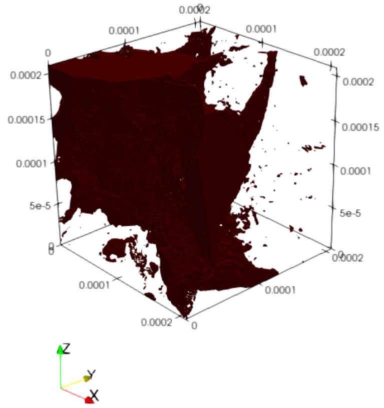
- nx = 210, ny = 210, nz = 200
- Lx = 0.25 mm, Lx = 0.25 mm, Lz = 0.24 mm
- dx = 1.2 microns, dy = 1.2 microns, dz = 1.2 microns



Porous Space Sample A



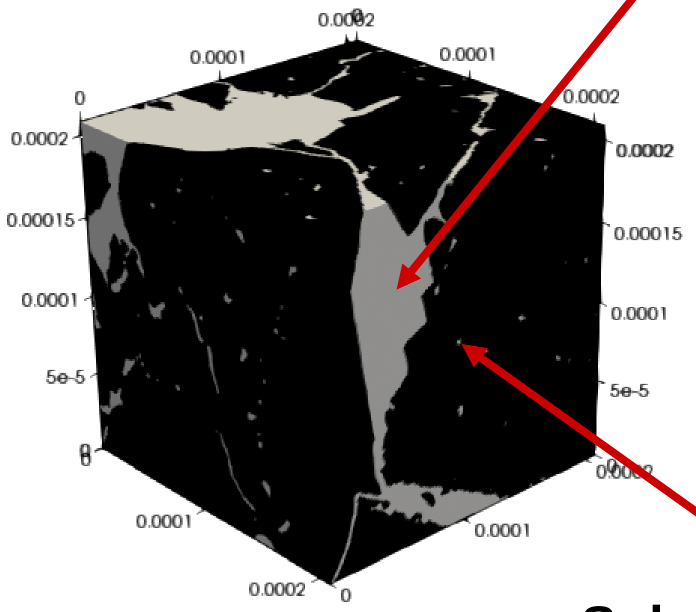
Porous Space Sample B



Numerical Procedure to Determine Full-Tensor Permeabilities

- We use the SIMPLE method to solve the steady-state Stokes-Brinkman Equations
- We use the algebraic multigrid method (AMG) to solve the momentum and pressure correction linear systems
- The systems are solved in parallel using the HYPRE library

Free flow region



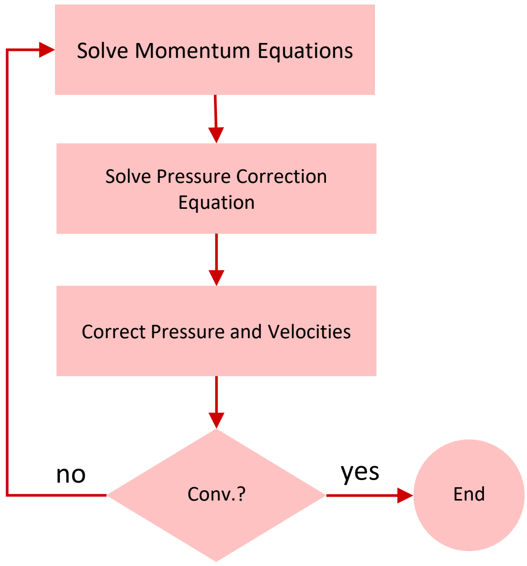
Sub-Resolution Porosity/Permeability

Stokes-Brinkman Model

$$-\mu\nabla^2\mathbf{u} + k^{-1}\mathbf{u} + \nabla p = 0$$

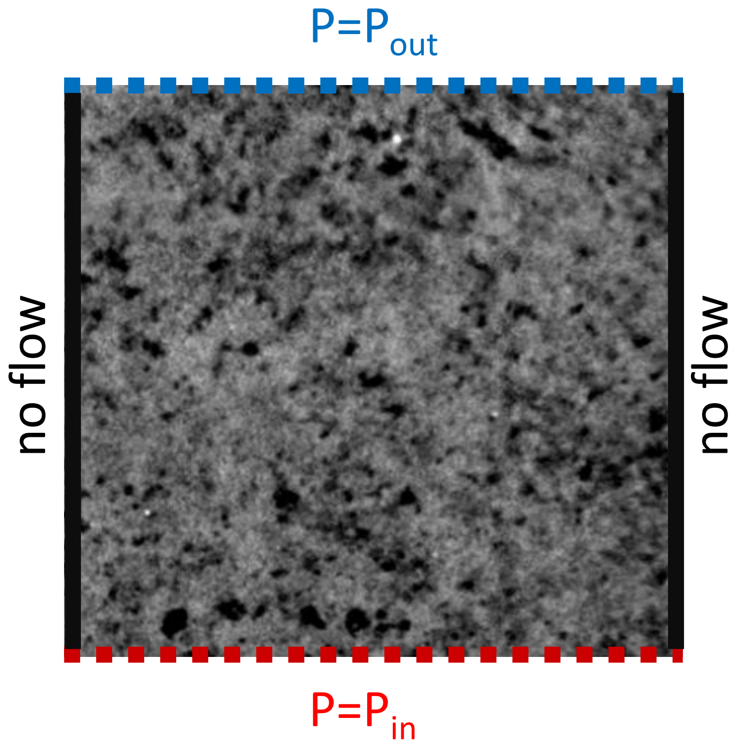
$$\nabla \cdot \mathbf{u} = 0$$

SIMPLE Method

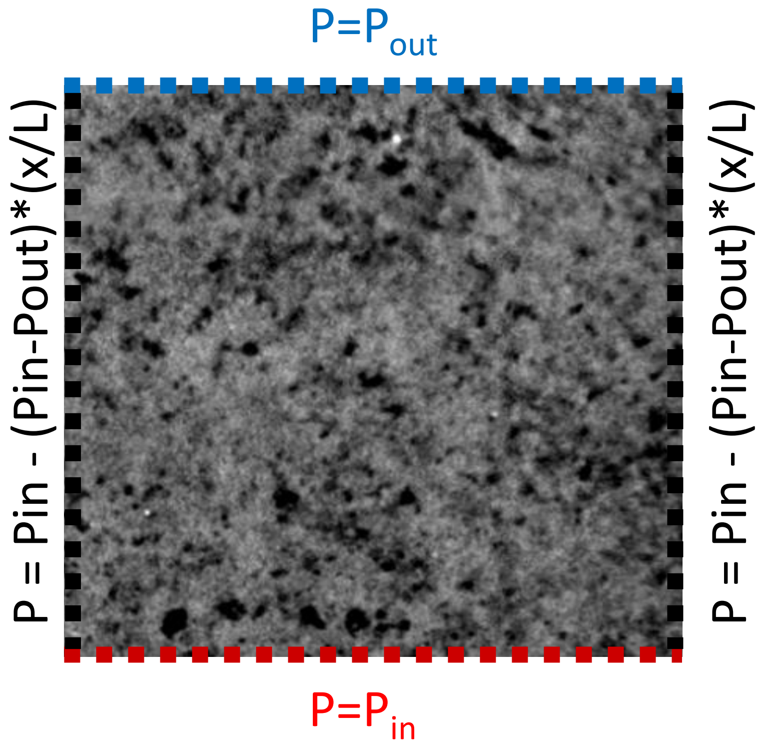


Boundary Conditions

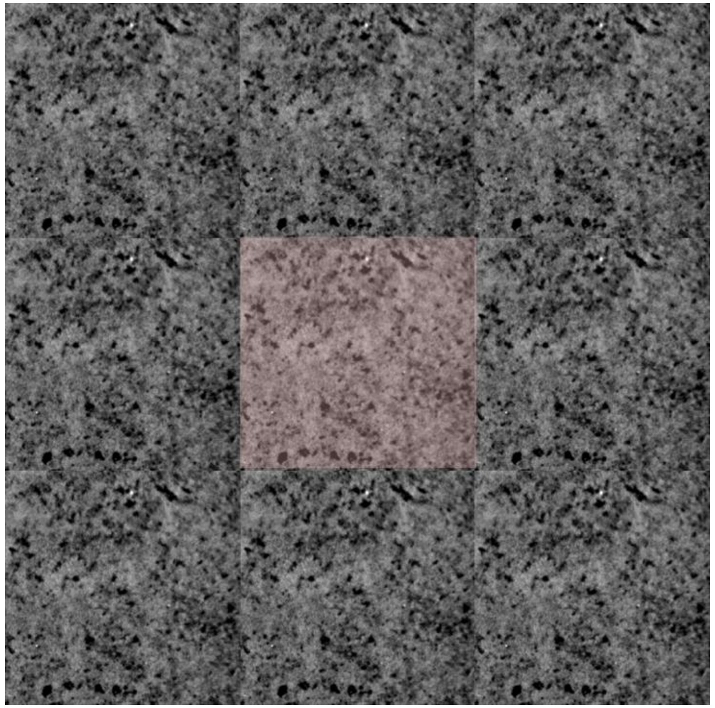
No-flow BCs
Diagonal permeability tensor



Linear Pressure BCs
Full permeability tensor **BUT** might not represent in-situ reservoir conditions



Periodic BCs
Full permeability tensor **BUT** might not be representative of a real rock



Results

Sample A | **Sample B**

Linear Pressure BCs

Sample A:

$$\mathbf{k}\ [\text{mD}] = \begin{pmatrix} 129 & 12 & \color{red}{46\ (35\%)} \\ & 113 & 12 \\ & & 114 \end{pmatrix}$$

Sample B:

$$\mathbf{k}\ [\text{mD}] = \begin{pmatrix} 1922 & 111 & \color{red}{486\ (25\%)} \\ & 1332 & 59 \\ & & 1243 \end{pmatrix}$$

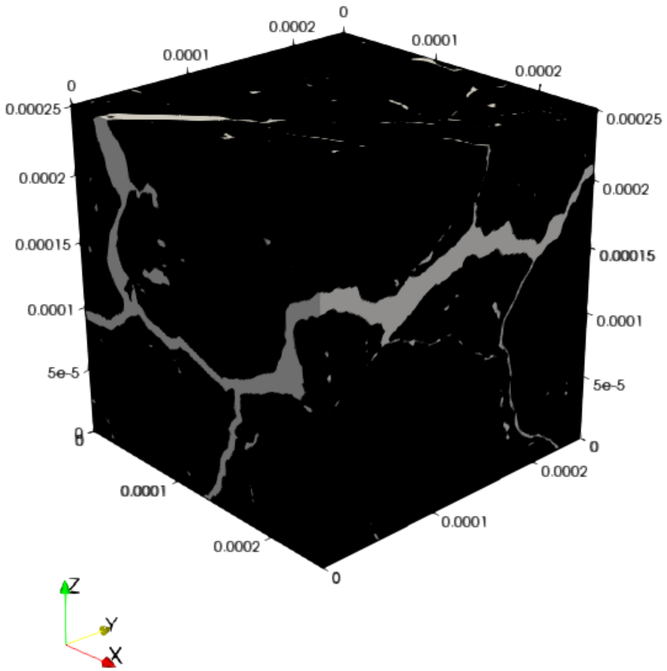
Periodic BCs

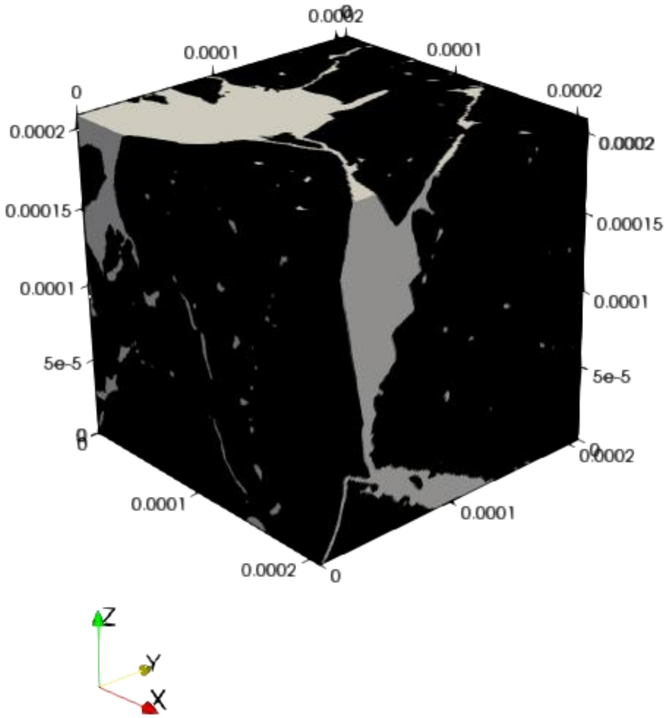
Sample A:

$$\mathbf{k}\ [\text{mD}] = \begin{pmatrix} 52 & 16 & \color{red}{22\ (42\%)} \\ & 95 & 0.05 \\ & & 40 \end{pmatrix}$$

Sample B:

$$\mathbf{k}\ [\text{mD}] = \begin{pmatrix} 763 & \color{red}{128\ (16\%)} & 107 \\ & 19 & 1 \\ & & 466 \end{pmatrix}$$





Upscaling to larger scales

- Typical reservoir simulators: Darcy flux between two cells are approximated by a transmissibility multiplied by a potential difference between the cells
- This assumes: the grid is aligned with the principal directions of the permeability tensor
- This method is typically called “Two-point flux approximation”
- The Multipoint Flux Approximation considers a larger stencil and uses more neighbor cells

TPFA

- Implemented in most reservoir simulators
- Considers only two cells for flux approximation
- Cannot have a full tensor as an input

vs

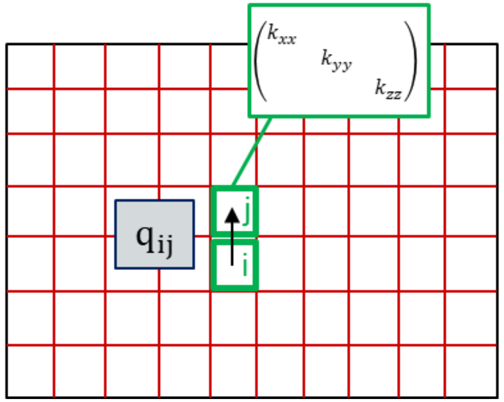
MPFA

- Complex to implement
- Considers a larger stencil for flux approximation
- Can have a full tensor as input

Present solver

Flux between two cells computed using two pressure values

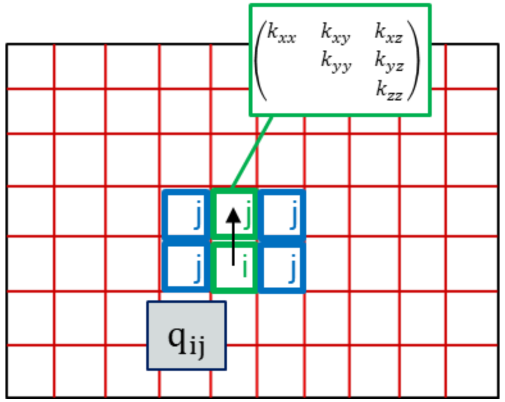
$$q_{ij} = T_{ij}(p_j - p_i)$$



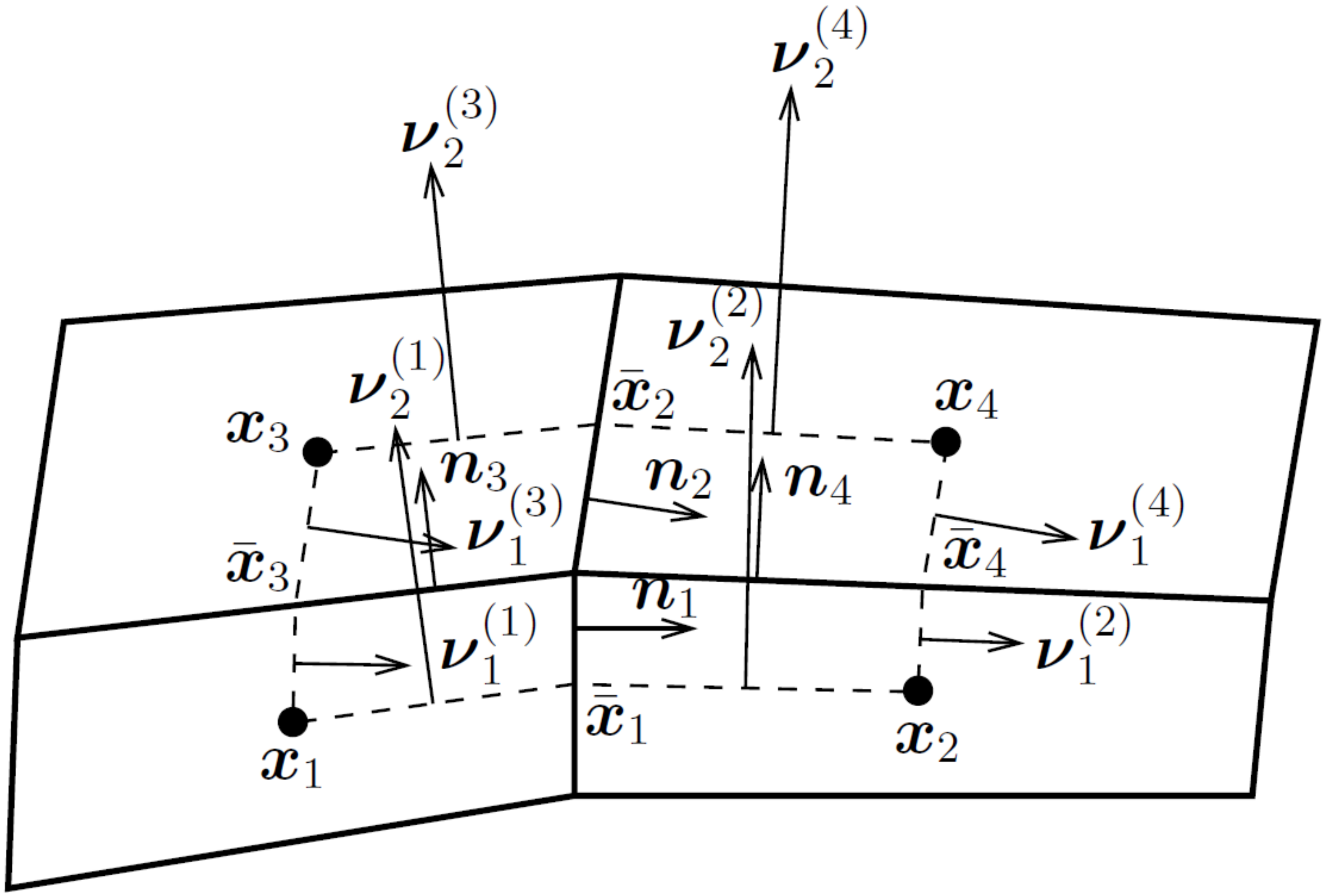
Improved solver

Flux between two cells computed using all neighboring cells

$$q_{ij} = \Sigma_j \tau_j (p_j - p_i)$$



MPFA Fundamentals



Transmissibility matrix

fluxes

pressure

$$\mathbf{f} = \mathbf{T}\mathbf{u}$$

$$\mathbf{T} = \mathbf{C}\mathbf{A}^{-1}\mathbf{B} + \mathbf{F}$$

Preliminary Results

- Validation in a small (5x5x5) 3D volume against manufactured analytical solution

$$\nabla \cdot k \nabla p = Q(x, y, z)$$

If we assume pressure is a given function, we can compute the derivatives and obtain the source:

$$p = e^{xyz} \Rightarrow Q(x, y, z) = e^{xyz} \cdot \begin{pmatrix} k_{xx} y^2 z^2 + \\ k_{yy} x^2 z^2 + \\ k_{zz} x^2 y^2 + \\ 2k_{xy}(z + xyz^2) + \\ 2k_{xz}(y + xy^2 z) + \\ 2k_{xy}(x + x^2 yz) \end{pmatrix}$$

Preliminary Results

- Validation in a small (5x5x5) 3D volume against manufactured analytical solution

<u>Case 1</u>: $\mathrm{k_{xx}} = 10^{-11}\ \mathrm{m}^2$, $\mathrm{k_{yy}} = 10^{-11}$, $\mathrm{k_{zz}} = 10^{-11}$ (isotropic, base case)

<u>Case 2</u>: $\mathrm{k_{xx}} = 10^{-11}\ \mathrm{m}^2$, $\mathrm{k_{yy}} = 10^{-13}$, $\mathrm{k_{zz}} = 10^{-11}$ (change of k in y dir.)

<u>Case 3</u>: $\mathrm{k_{xx}} = 10^{-11}\ \mathrm{m}^2$, $\mathrm{k_{yy}} = 10^{-11}$, $\mathrm{k_{zz}} = 10^{-12}$ (change of k in z dir)

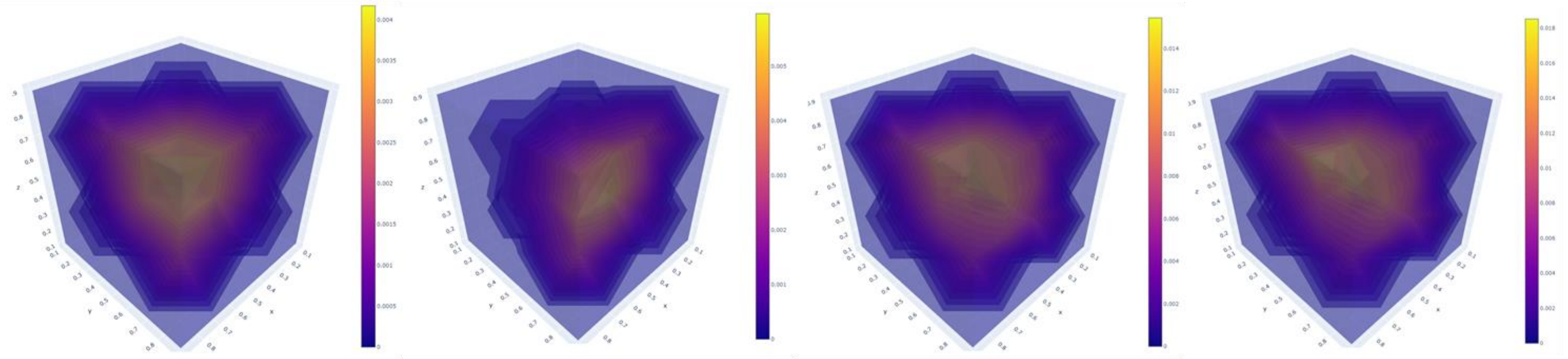
<u>Case 4</u>: $\mathrm{k_{xx}} = 10^{-11}\ \mathrm{m}^2$, $\mathrm{k_{yy}} = 10^{-11}$, $\mathrm{k_{zz}} = 10^{-13}$ (change of k in z dir)

<u>Case 1</u> $\|(\mathrm{p} - \mathrm{p_{an}})/\mathrm{p}\|_\infty \approx 0.4\%$

<u>Case 2</u> $\|(\mathrm{p} - \mathrm{p_{an}})/\mathrm{p}\|_\infty \approx 0.6\%$

<u>Case 3</u> $\|(\mathrm{p} - \mathrm{p_{an}})/\mathrm{p}\|_\infty \approx 1.4\%$

<u>Case 4</u> $\|(\mathrm{p} - \mathrm{p_{an}})/\mathrm{p}\|_\infty \approx 1.8\%$

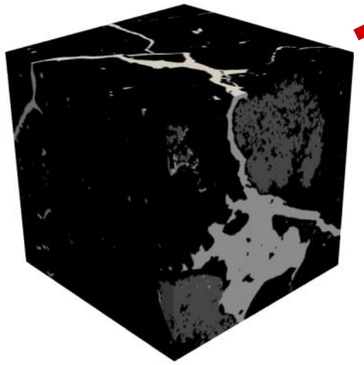


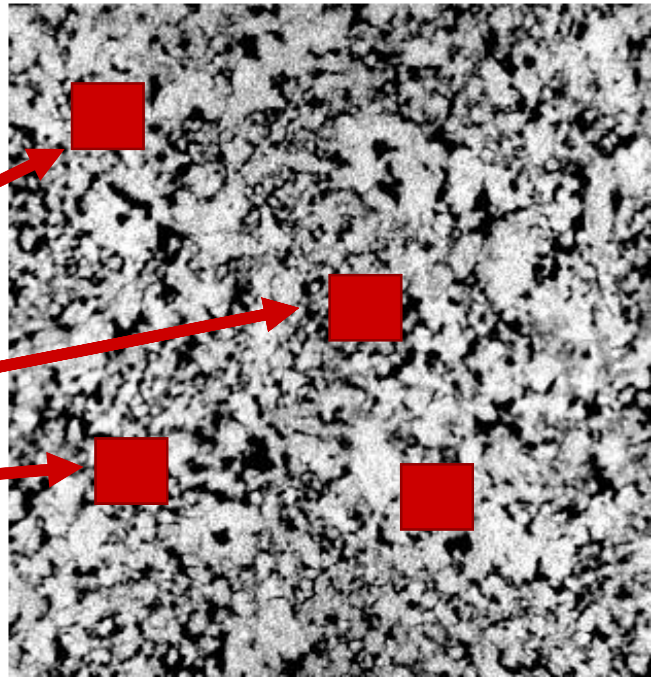
Next Step

?

$$\boldsymbol{k} = \begin{pmatrix} \boldsymbol{k_{xx}} & k_{xy} & k_{xz} \\ sym. & \boldsymbol{k_{yy}} & k_{yz} \\ sym. & sym. & \boldsymbol{k_{zz}} \end{pmatrix}$$

$$\mathbf{k}\ [\mathrm{mD}] = \begin{pmatrix} 129 & 12 & 46 \\ & 113 & 12 \\ & & 114 \end{pmatrix}$$





THANK YOU!