# Introduction into Semi-Classical Analysis

Sergey Sergeev
sergeevse1@mat.puc-rio.br

This course is aimed to present some of the basic theory of semi-classical asymptotic analysis in partial differential equations. The semi-classical analysis is based on the special form of fast-oscillation exponential (WKB form)

$$A(x,\, t)e^{iS(x,\, t)/h}, \quad h \ll 1$$

for the solution. This type of the form for the solution was implemented in the Schrödinger equation. It is known that near the turning points this representation is not valid and other representation has to be found. This can be done with the help of Maslov's Canonical Operator.

We will discuss the 1D spectral problem for the 1D Schrödinger equation and describe it's semi-classical asymptotic solution for large values of eigenvalues. Another application for the semi-classical analysis which will be discused is the asymptotic description of the linear surface wave propagation for far distances.

The program for the course is as follows

1. 1D sepctral problem for the Schrödinger equation and WKB substitution
2. Representation near the turning points
3. Definition of the Maslov's Canonical Operator in 1D case
4. 2D Cauchy problem for the surface linear wave propagation over non-uniform bottom
5. Pseudo-differential operators
6. Definition of the Maslov's Canonical Operator in 2D case
7. Lagrangian manifold and elements of simplectic geometry
8. Asymptotic description of the 2D surface wave

**Preliminaries:**
1) Ordinary differential equations
2) Partial differential equations

**Literature:**

1) M. Zworski, Semiclassical analysis. // AMS, Graduate Studies in Mathematics, Volume 138, 2012

2) V.P. Maslov, M.V. Fedoriuk, Semi-Classical Approximation in Quantum Mechanics. //Springer, Softcover reprint of the original 1st ed. 1981 edition (November 30, 2001)

3) V.E. Nazaikinskii, B.-W. Schulze, B. Yu. Sternin, Quantization Methods in Differential Equations. // CRC Press, 2002

4) L. Hörmander, The analysis of Linear Partial Differential Operators III and IV

**Timetable:** Two times per week starting from the January 4-th till February 27-th by Tuesdays and Thursdays from 13-00 till 16-00 hours.