MAT2832 Grigory Ivanov

1. Asymptotic Geometric Analysis

1.1. **Description.** The goal of this course is to explore several paradoxical properties of convex bodies as functions of dimensionality. We will begin with classical results in convexity, such as the Brunn–Minkowski inequality and its numerous corollaries, the Brascamp–Lieb inequality, and various forms of isoperimetric inequalities. We will also examine several classical positions of convex bodies. The primary focus will be to introduce and utilize the concentration of measure phenomenon as the main tool of the course.

We will then apply this tool to a variety of problems. For instance, we will explore how the Johnson–Lindenstrauss flattening lemma can exponentially reduce dimensionality in certain scenarios, study the distribution of volume in high-dimensional convex bodies, and prove the Dvoretzky–Milman theorem. Time permitting, we will also discuss how these results relate to problems from other areas, such as the intriguing Kadison–Singer problem.

1.2. **Topics.**

(1) Basic convexity
(2) Brunn–Minkowski inequality
(3) Mass transportation problem
(4) Brascamp–Lieb inequality and its reverse form
(5) Positions of convex bodies
(6) Isoperimetric inequalities
(7) Deviation inequalities for Lipschitz functions
(8) Concentration of measure phenomenon
(9) Fine approximations of convex bodies
(10) Johnson–Lindenstrauss flattening lemma
(11) Almost Euclidean subspaces of finite-dimensional normed spaces
(12) Khintchine-type inequalities
(13) Sparsification of matrices and convex bodies

1.3. **Prerequisites.**

- Mathematical analysis
- Linear algebra
- Basic knowledge of convex analysis
- Basic knowledge of functional analysis

References

[1] S. Artstein-Avidan, A. Giannopoulos, and V. D. Milman. *Asymptotic geometric analysis, Part I*, volume 202. American Mathematical Soc., 2015.

[2] G. Aubrun and S. J. Szarek. *Alice and Bob Meet Banach: The Interface of Asymptotic Geometric Analysis and Quantum Information Theory*, volume 223. American Mathematical Soc., 2017.

[3] S. Brazitikos, A. Giannopoulos, P. Valettas, and B.-H. Vritsiou. *Geometry of isotropic convex bodies*, volume 196. American Mathematical Society Providence, 2014.