Geometry of Algebraic Varieties

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Place, time, schedule **DMAT PUC-Rio**, **2024.2**, **Tuesdays** and **Thursdays**, **15h – 17h.** *It is possible to change Thursday to Friday morning (before 13h), let me know if this is preferred option.* Problem assignment: from June 10 to August 24. Public discussion of solutions: from Aug 27 to Dec 13. Holidays: Oct 15 (Tue) and Nov 15-20 (Fri,...,Wed). So 2 or 3 days out of 16 weeks (and 32 classes) are missing, making total of 29-30 days for discussion of problems and prerequisites.

Target audience Students (at least minimally) introduced either to Algebraic Geometry (e.g. MAT2255) or to Complex Geometry (e.g. MAT2256). *Students that only had advanced Riemann Surfaces (such as MAT2816) or Algebraic Curves, or basic Projective Geometry shall try at their own risk: there are plenty of good problems about algebraic curves, but they may have difficulties in appreciating solutions of others.*

Plan Every student in the beginning of the course is assigned a personalized set of 15 problems (to be selected together by student and instructors) to solve and explain to others during the semester. Solutions of problems shall take at least half of the time, with formulations and necessary explanations or background taking the rest. Final grade can be set to any P (at most 10), for which a student delivered at least P problems to their peers and instructors (Sergey and Lucas), and served at least at 2P sessions as a peer. One student is allowed to deliver at most one problem per day.

Bibliography Two main references with exercises for algebraic and complex geometry used to be

- Robin Hartshorne: Algebraic Geometry
- Philippe Griffiths and Joe Harris: Principles of Algebraic Geometry

In particular, it is a well-known feature of Hartshorne's textbook that some important parts of the theory are delegated to exercises, so students who skipped them might not digested the theory.

Problems from other textbooks, such as Huybrechts's "Complex geometry", Voisin's "Hodge theory and complex algebraic geometry", Vakil's "The rising sea: foundations of algebraic geometry", Mukai's "Introduction to invariants and moduli", Shafarevich's "Basic algebraic geometry", Harris's "Algebraic geometry: start up course", Liu's "Algebraic geometry and arithmetic curves", and more specialized books as Mumford's "Abelian varieties", Beauville's "Algebraic surfaces", based on students' background and intentions. So feel free to suggest your bibliography.

Instructors and students will provide some bonus problems from the pool of their favourite. E.g.: show or disprove that any algebraic variety is dominated by a product of curves. (here variety assumed to be irreducible, and X is said to be dominated by Y if there exists a dominant rational map from Y to X).