MAT2725

Professor: Sergei Burkin

Stable Homotopy Theory

We will study generalized cohomology theories via the modern approach to spectra. A generalized cohomology theory is a functor from spaces to graded R-modules that satisfies all Eilenberg-Steenrod axioms except the dimension axiom. Examples include topological K-theory, cobordism and stable homotopy groups. Any generalized cohomology theory is represented by a structure called spectrum. Spectra is what you get from spaces once you try to make invertible the suspension and the loop space functors. There are several equivalent approaches to spectra. The approaches that allow to encode multiplication in generalized cohomology in straightforward way became available only in the beginning of this century. This led to several breakthrough results. Our aim is both to learn this modern language and to formulate classical constructions and applications (Hopf invariant one problem, vector fields on spheres, computation of stable homotopy groups, classification of manifolds up to cobordism). For more details about what we might cover see [2], [3] and [1].

Prerequisites: simplicial (co)homology and some familiarity with homotopy theory (homotopy groups, (co)fibrations, Dold-Puppe sequence, Eilenberg-MacLane spaces, Postnikov and Whitehead towers, Freudenthal suspension theorem, representability of ordinary cohomology). Familiarity with spectral sequences and with model categories is not assumed.

Literature:
1. J.F. Adams. Stable homotopy and generalised homology. 1974. (Classical textbook. We need only part III)

2. Stanley O Kochman. Bordism, stable homotopy and Adams spectral sequences. 1996.

3. Akira Kono, Dai Tamaki. Generalized cohomology. 2006.
4. Barnes, Roitzheim. Foundations of stable homotopy theory. 2020. (Good for learning the language, but does not give enough applications)

5. Douglas Ravenel. Complex cobordism and stable homotopy groups of spheres. 2nd ed. 2004. (More advanced)

6. Mosher, Tangora. Cohomology operations and applications in homotopy theory. 1968. (Useful for learning some background material)