

## ADVANCED TOPICS IN THE ARITHMETIC OF ELLIPTIC CURVES

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We propose to give a tour of several advanced topics in the arithmetic of elliptic curves, leading to a full description of the theory of elliptic curves with complex multiplication. This will begin with some relevant background from Silverman [6], [7] on the arithmetic of elliptic curves (e.g. over local fields and number fields), then focus on the special case of elliptic curves with complex multiplication. In this direction, we shall first prove the classical theorem of Hecke and Deuring about the analytic continuation of the Hasse-Weil  $L$ -function, then review the theory of complex multiplication ([5], [4]), leading to some description of the so-called Iwasawa main conjectures shown by Coates-Wiles ([2], [3]). This development will begin with a sustained discussion of the special values of the Hasse-Weil  $L$ -functions, including their realizations as special values of Eisenstein series ([8]), the rationality theorem of Damerrell, and the construction of  $p$ -adic interpolation series. The course will be given in tandem with a Friday-morning graduate student seminar on the more advanced Iwasawa theoretic aspects.

*Prerequisites.* Algebra (including Galois theory), real and complex analysis, and ideally some exposure to number theory – particularly algebraic number fields and  $p$ -adic local fields – as well as elliptic curves (e.g. over the complex numbers).

### REFERENCES

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- [2] J. Coates and A. Wiles. *On the Conjecture of Birch and Swinnerton-Dyer*. Invent. math. **39** (1977), 223-251.
- [3] E. de Shalit. *Iwasawa Theory of Elliptic Curves with Complex Multiplication*. Perspectives in Mathematics **3** Academic Press London (1987).
- [4] J. Neukirch. *Algebraische Zahlentheorie*. Springer-Verlag Berlin (1992).
- [5] J.-P. Serre. *Complex Multiplication*. in "Algebraic Number Theory", Eds. J. W. S. Cassels and A. Fröhlich, Academic Press, London, 1967.
- [6] J. Silverman. *The Arithmetic of Elliptic Curves*. Springer GTM **106**. Springer-Verlag New York Inc. (1986).
- [7] J. Silverman. *Advanced Topics in the Arithmetic of Elliptic Curves*. Springer GTM **151**. Springer-Verlag, New York Inc. (1991).
- [8] A. Weil. *Elliptic functions according to Eisenstein and Kronecker*. Classics in Mathematics. Springer, Berlin Heidelberg New York (1976).