FOURIER ANALYSIS ON GROUPS: PONTRYAGIN DUALITY AND APPLICATIONS

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Scope. This course develops the classical theory of Fourier analysis on locally compact abelian (LCA) groups, with Pontryagin duality as the central organizing principle. The first part follows the foundational chapters of [3], emphasizing Haar measure, Fourier transform, inversion, Plancherel, and positive-definite functions. The second part presents selected modern developments that build directly on this toolkit: profinite and non-archimedean examples, random walks on compact/profinite groups, and a guided glimpse of the Plancherel formula in the non-abelian direction via the abelian model GL_1 and statement-level context for GL_n.

Prerequisites. Measure-theoretic probability and functional analysis basic harmonic analysis on $\mathbf{R}^2$, and topological groups (basic knowledge). No prior exposure to LCA groups required.

Topics/outline. We start with the main theorems of Fourier analysis following the first few chapters of [3], then present some modern developments.

- The main theorems of Fourier analysis: Haar measure and convolution, dual groups and Fourier transforms, the Fourier-Stieltjes transform, positive-definite functions and Bochener's theorem the inversion theorem, the Plancherel theorem, Pontryagin duality, Bohr compactification, and $B(\Gamma)$
- The structure of locally compact abelian (LCA) groups, idempotent measures, homomorphisms of group algebras, thin sets, closed ideals and subalgebras of $L^1(G)$.
- Special topics: Profinite groups, random walks on groups, abstract representation theory and the Plancherel theorem for $\mathrm{GL}(n)$.

Evaluation.

- Problem sets (every 2-3 weeks), 40%
- Mini-project (short exposition or computational experiment), 20 %
- Final oral exam (with written report), 60 %

References

[1] Y. Benoit and J.-F. Quint, *Random walks on reductive groups*, Springer Nature (2016)
[2] J.-F. Quint, *An introduction to random walks on groups*, online course notes
[3] W. Rudin, *Fourier Analysis on Groups*, Wiley (1962)
[4] N. Wallach, *Real reductive groups, II*, Academic Press (1992)