SPECTRAL METHODS OF AUTOMORPHIC FORMS AND ANALYTIC NUMBER THEORY

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Scope. The course will develop the spectral theory of automorphic forms, emphasizing the spectral decomposition of $L^2(\Gamma\backslash\mathfrak{H})$, the Selberg trace formula, the Kuznetsov trace formula, estimates for Fourier coefficients of Maass forms, approaches to the shifted convolution problem, and applications to analytic number theory.

Prerequisites. Complex analysis, basic algebraic number theory (Dirichlet characters and ideal class groups). Ideally, the students will have seen some representation theory, modular forms, and functional analysis.

Topics/outline. We propose to follow the treatment in [2], with additional discussion from [3] and [4].

- Harmonic analysis on the hyperbolic plane: the upper-half plane $\mathfrak{H}$, group decompositions, motions, the Laplace operator and its eigenfunctions, invariant integral operators, Green's functions.
- Fuchsian groups and automorphic forms: Definitions, fundamental domains, examples, double coset decompositions, Kloosterman sums, Eisenstein series, cusp forms.
- The spectral theorem: The discrete spectrum, analytic continuation of Eisenstein series, the continuous spectrum.
- Estimates for Fourier coefficients of Maass forms: Spectral mean value estimates and spectral decompositions of shifted convolution sums, Rankin-Selberg L-functions.
- Spectral theory of Kloosterman sums and the Kuznetsov trace formula.
- The Selberg trace formula, the Selberg Zeta function and lengths of closed geodesics.
- Other applications to number theory: e.g. moments of automorphic L-functions, QUE, automorphic periods, equidistribution results.

Evaluation.

- Problem sets (every 2-3 weeks), 40%
- Mini-project (short exposition or computational experiment), 20 %
- Final oral exam (with written report), 60 %

References

[1] A. Granville and Z. Rudnick (eds.), *Equidistribution in Number Theory, An Introduction*, NATO Science Series (2007).
[2] H. Iwaniec, *Spectral Methods of Automorphic Forms*, Grad. Stud. Math. **53**, Amer. Math. Soc., Providence RI (2002).
[3] H. Iwaniec, *Topics in Classical Automorphic Forms*, Grad. Stud. Math. **17**, Amer. Math. Soc., Providence RI (1997).
[4] H. Iwaniec and E. Kowalski, *Analytic Number Theory*, Amer. Math. Soc. Colloq. Publ. **53** AMS Providence (2004).