Necessary conditions for dynamical coherence

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partial hyperbolicity

Definition

$f : M^3 \to M^3$ is partially hyperbolic

$$TM = E^s \oplus E^c \oplus E^u$$

partial hyperbolicity

Definition

$f : M^3 \to M^3$ is partially hyperbolic

$$TM = \underset{\substack{\uparrow \\ \text{contracting}}}{E^s} \oplus \underset{\substack{\uparrow \\ \text{intermediate}}}{E^c} \oplus \underset{\substack{\uparrow \\ \text{expanding}}}{E^u}$$

partial hyperbolicity

Definition

$f : M^3 \to M^3$ is partially hyperbolic

$$TM = \underset{\substack{\uparrow \\ \text{1-dim}}}{E^s} \oplus \underset{\substack{\uparrow \\ \text{1-dim}}}{E^c} \oplus \underset{\substack{\uparrow \\ \text{1-dim}}}{E^u}$$

integrability

- $TM = E^s \oplus E^c \oplus E^u$

integrability

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- $\exists!$ invariant foliation $\mathcal{F}^s$ tangent to E^s

integrability

- $TM = E^s \oplus E^c \oplus E^u$
- $\exists!$ invariant foliation $\mathcal{F}^s$ tangent to E^s
- $\exists!$ invariant foliation $\mathcal{F}^u$ tangent to E^u

integrability

- $TM = E^s \oplus E^c \oplus E^u$
- $\exists!$ invariant foliation $\mathcal{F}^s$ tangent to E^s
- $\exists!$ invariant foliation $\mathcal{F}^u$ tangent to E^u
- $\exists$ invariant foliation tangent to E^c?

dynamical coherence

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- $f : M \to M$ partially hyperbolic

dynamical coherence

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- is dynamically coherent if

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  1. $\exists$ invariant foliation $\mathcal{F}^{cs}$ tangent to $E^s \oplus E^c$

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  1. $\exists$ invariant foliation $\mathcal{F}^{cs}$ tangent to $E^s \oplus E^c$
  2. $\exists$ invariant foliation $\mathcal{F}^{cu}$ tangent to $E^c \oplus E^u$

dynamical coherence

dynamical coherence

- $f : M \to M$ partially hyperbolic
- is dynamically coherent if
  1. $\exists$ invariant foliation $\mathcal{F}^{cs}$ tangent to $E^s \oplus E^c$
  2. $\exists$ invariant foliation $\mathcal{F}^{cu}$ tangent to $E^c \oplus E^u$
  3. $\Rightarrow \exists$ invariant foliation $\mathcal{F}^c$ tangent to E^c

theorem

theorem (Brin-Burago-Ivanov)

If $f : \mathbb{T}^3 \to \mathbb{T}^3$ is absolutely partially hyperbolic, then f is dynamically coherent.

question

question

If $f : M^3 \to M^3$ is partially hyperbolic, is f dynamically coherent?

question

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If $f : M^3 \to M^3$ is partially hyperbolic, is f dynamically coherent?
NO

theorem

theorem (HHU)

There exists $f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic such that

theorem

theorem (HHU)

There exists $f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic such that

- f is not dynamically coherent

theorem

theorem (HHU)

There exists $f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic such that

- f is not dynamically coherent
- f is robustly non-dynamically coherent

conjecture

conjecture [HHU]

All conservative partially hyperbolic diffeomorphisms of a 3-manifold are dynamically coherent.

necessary conditions for dynamical coherence

main theorem (HHU)

- $\mathcal{F}^{cu}$ invariant foliation

necessary conditions for dynamical coherence

main theorem (HHU)

- $\mathcal{F}^{cu}$ invariant foliation
- $\mathcal{F}^{cu}$ tangent to $E^c \oplus E^u$

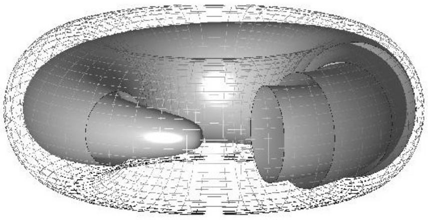
necessary conditions for dynamical coherence

main theorem (HHU)

- $\mathcal{F}^{cu}$ invariant foliation
- $\mathcal{F}^{cu}$ tangent to $E^c \oplus E^u$
- $\Rightarrow \mathcal{F}^{cu}$ does not contain compact leaves

Reeb component

Reeb component



general remark

remark 1

- $\mathcal{F}^{\pitchfork}$ 1-dimensional foliation

general remark

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- $\mathcal{F}^{\pitchfork}$ 1-dimensional foliation
- $\mathcal{F}^{\pitchfork}$ transverse to a Reeb component

general remark

remark 1

- $\mathcal{F}^{\pitchfork}$ 1-dimensional foliation
- $\mathcal{F}^{\pitchfork}$ transverse to a Reeb component
- $\Rightarrow \mathcal{F}^{\pitchfork}$ contains a closed loop

dynamical coherence and Reeb components

remark 2

- $\mathcal{F}_{\varepsilon}^{cu}$ almost tangent to $E^c \oplus E^u$

dynamical coherence and Reeb components

remark 2

- $\mathcal{F}^{cu}_{\varepsilon}$ almost tangent to $E^c \oplus E^u$
- $\Rightarrow \mathcal{F}^{cu}$ does not have Reeb components

dynamical coherence and Reeb components

remark 2

- $\mathcal{F}^{cu}_{\varepsilon}$ almost tangent to $E^c \oplus E^u$
- $\Rightarrow \mathcal{F}^{cu}$ does not have Reeb components

proof

remark 1

remark

property of codimension-one foliations

- $\tilde{M}$ simply connected

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- $\mathcal{F}^{cu}$ codimension-one foliation

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property of codimension-one foliations

- $\tilde{M}$ simply connected
- $\mathcal{F}^{cu}$ codimension-one foliation
- $\exists$ closed loop transverse to $\mathcal{F}^{cu}$

remark

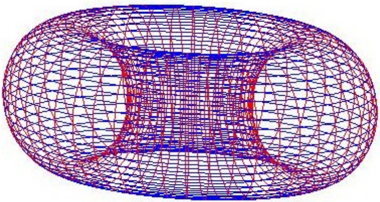
property of codimension-one foliations

- $\tilde{M}$ simply connected
- $\mathcal{F}^{cu}$ codimension-one foliation
- $\exists$ closed loop transverse to $\mathcal{F}^{cu}$
- $\Rightarrow \mathcal{F}^{cu}$ has a Reeb component

Anosov torus

Anosov torus

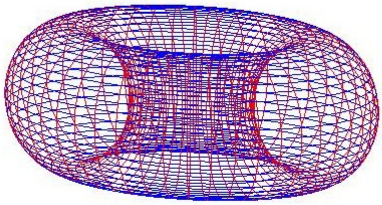
- T embedded 2-torus



Anosov torus

Anosov torus

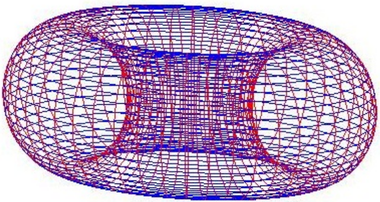
- T embedded 2-torus
- T Anosov torus



Anosov torus

Anosov torus

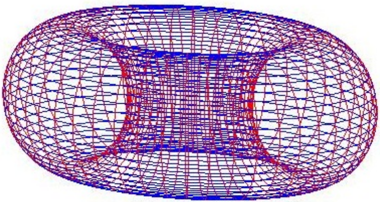
- T embedded 2-torus
- T Anosov torus
- if $\exists f : M \to M$ s.t.



Anosov torus

Anosov torus

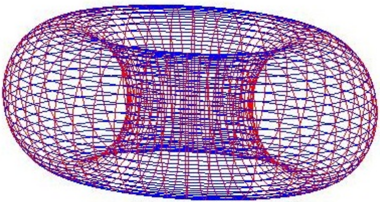
- T embedded 2-torus
- T Anosov torus
- if $\exists f : M \to M$ s.t.
  1. $f(T) = T$



Anosov torus

Anosov torus

- T embedded 2-torus
- T Anosov torus
- if $\exists f : M \to M$ s.t.
  1. $f(T) = T$
  2. $f|_T$ isotopic to Anosov



theorem

theorem (HHU)

If an irreducible M contains an Anosov torus,

theorem

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If an irreducible M contains an Anosov torus, then M is either

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1. $\mathbb{T}^3$

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2. the mapping torus of $-id : \mathbb{T}^2 \to \mathbb{T}^2$

theorem

theorem (HHU)

If an irreducible M contains an Anosov torus, then M is either

1. $\mathbb{T}^3$
2. the mapping torus of $-id : \mathbb{T}^2 \to \mathbb{T}^2$
3. a mapping torus of a hyperbolic automorphism of $\mathbb{T}^2$

remark

remark

If $f : M^3 \to M^3$ partially hyperbolic, then

remark

remark

If $f : M^3 \to M^3$ partially hyperbolic, then

- M is irreducible (Burago-Ivanov08, Rosenberg68)

reduced theorem

reduced theorem

- N^3 irreducible manifold with boundary

reduced theorem

reduced theorem

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- ∂N consists of Anosov tori

reduced theorem

reduced theorem

- N^3 irreducible manifold with boundary
- ∂N consists of Anosov tori
- $\Rightarrow$

$$N = \mathbb{T}^2 \times [0, 1]$$

structure of the proof

step 1

- $\exists$ a compact leaf in $\mathcal{F}^{cu}$

structure of the proof

step 1

- $\exists$ a compact leaf in $\mathcal{F}^{cu}$
- $\Rightarrow \exists$ an Anosov torus T in $\mathcal{F}^{cu}$

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- $\exists$ a compact leaf in $\mathcal{F}^{cu}$
- $\Rightarrow \exists$ an Anosov torus T in $\mathcal{F}^{cu}$

step 2

- $\exists\ h : N \to T$ semiconjugacy between f and A

structure of the proof

step 1

- $\exists$ a compact leaf in $\mathcal{F}^{cu}$
- $\Rightarrow \exists$ an Anosov torus T in $\mathcal{F}^{cu}$

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- $\exists\ h : N \to T$ semiconjugacy between f and A
- h takes W^c_f to W^s_A in T

structure of the proof

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- $\exists$ a compact leaf in $\mathcal{F}^{cu}$
- $\Rightarrow \exists$ an Anosov torus T in $\mathcal{F}^{cu}$

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- $\exists\ h : N \to T$ semiconjugacy between f and A
- h takes W^c_f to W^s_A in T

step 3

There exists a closed loop transverse to $\tilde{\mathcal{F}}^{cu}$ in $\tilde{M}$

structure of the proof - let's begin

hypothesis

- $\mathcal{F}^{cu}$ foliation

structure of the proof - let's begin

hypothesis

- $\mathcal{F}^{cu}$ foliation
- $\exists$ compact leaf

structure of the proof - let's begin

hypothesis

- $\mathcal{F}^{cu}$ foliation
- $\exists$ compact leaf

goal

reach a contradiction

step 1

step 1

there exists an Anosov torus T in $\mathcal{F}^{cu}$

step 1

proposition

there exists a compact periodic leaf T in $\mathcal{F}^{cu}$

step 1

- $\Lambda = \{x : x \text{belongs to a compact leaf}\}$

step 1

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- Λ is compact (Häfliger)

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- E^s is transverse to T
- E^s uniformly contracting

step 1

- $\Lambda = \{x : x \text{belongs to a compact leaf}\}$
- Λ is compact (Häfliger)
- $\Rightarrow \exists$ a recurrent compact leaf T
- E^s is transverse to T
- E^s uniformly contracting
- $\Rightarrow$ T is periodic

step 1

assume $f(T) = T$

step 1

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proposition

$f_* : \pi_1(T) \to \pi_1(T)$ is hyperbolic

step 1

- f preserves $\mathcal{F}^u$

step 1

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- $f_* : \mathbb{Z}^2 \to \mathbb{Z}^2$ has eigenspace with irrational slope

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- if $f_* = id$ then $\tilde{f} = id$+periodic

step 1

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- $f_* : \mathbb{Z}^2 \to \mathbb{Z}^2$ has eigenspace with irrational slope
- $\Rightarrow$ either f_* hyperbolic or $f_* = id$
- if $f_* = id$ then $\tilde{f} = id$+periodic
- $\Rightarrow \text{diam}(\tilde{f}^n(W^u_\varepsilon(x))) \leq \text{diam}(W^u_\varepsilon(x)) + nK$

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- $\Rightarrow \exists$ arbitrarily long u-leaf with close endpoints

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- $\to$ CONTRADICTION

step 2

step 2

- $\exists\ h : N \to T$ semiconjugacy between f and A

step 2

step 2

- $\exists\ h : N \to T$ semiconjugacy between f and A
- h takes W_f^c to W_A^s in T

step 2

- cut M along T (Anosov torus)

step 2

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- obtain $N = T \times I$

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- obtain $N = T \times I$

Franks

$$\begin{array}{ccccc} & T \times I & \xrightarrow{f} & T \times I & \\ h & \downarrow & & \downarrow & h \\ & T & \xrightarrow{A} & T & \end{array}$$

step 2

- cut M along T (Anosov torus)
- obtain $N = T \times I$

Franks

$$
\begin{array}{cccccc}
 & T \times I & \xrightarrow{f} & T \times I & \\
h & \downarrow & & \downarrow & h \\
 & T & \xrightarrow{A} & T &
\end{array}
$$

- $h \circ f = A \circ h$

step 2

- cut M along T (Anosov torus)
- obtain $N = T \times I$

Franks

$$\begin{array}{ccccc} & T \times I & \xrightarrow{f} & T \times I & \\ h & \downarrow & & \downarrow & h \\ & T & \xrightarrow{A} & T & \end{array}$$

- $h \circ f = A \circ h$
- h homotopic to $\pi : T \times I \to T$

step 2

- cut M along T (Anosov torus)
- obtain $N = T \times I$

Franks

$$\begin{array}{ccccccc} & T \times I & \xrightarrow{f} & T \times I & \\ h & \downarrow & & \downarrow & h \\ & T & \xrightarrow{A} & T & \end{array}$$

- $h \circ f = A \circ h$
- h homotopic to $\pi : T \times I \to T$
- $h(T \times 0) = T$

step 2

proposition

h takes W^c_f into W^s_A on T

step 2

- $\gamma^c \subset T$ small

step 2

- $\gamma^c \subset T$ small
- $\varepsilon > 0$ small

step 2

- $\gamma^c \subset T$ small
- $\varepsilon > 0$ small
- $\Rightarrow W^u_\varepsilon(f^n(\gamma^c)) \subset f^n(W^u_\varepsilon(\gamma^c))$ for $n >> 0$

step 2

- $\gamma^c \subset T$ small
- $\varepsilon > 0$ small
- $\Rightarrow W^u_\varepsilon(f^n(\gamma^c)) \subset f^n(W^u_\varepsilon(\gamma^c))$ for $n >> 0$
- if length$(f^n(\gamma^c)) \to \infty$ then area$(W^u(f^n(\gamma^c))) \to \infty$

step 2

- $\gamma^c \subset T$ small
- $\varepsilon > 0$ small
- $\Rightarrow W^u_\varepsilon(f^n(\gamma^c)) \subset f^n(W^u_\varepsilon(\gamma^c))$ for $n >> 0$
- if length$(f^n(\gamma^c)) \to \infty$ then area$(W^u(f^n(\gamma^c))) \to \infty$
- (bounded angles)

step 2

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- if length$(f^n(\gamma^c)) \to \infty$ then area$(W^u(f^n(\gamma^c))) \to \infty$
- (bounded angles)
- $\to$ CONTRADICTION

step 2

- $\gamma^c \subset T$ small
- $\varepsilon > 0$ small
- $\Rightarrow W^u_\varepsilon(f^n(\gamma^c)) \subset f^n(W^u_\varepsilon(\gamma^c))$ for $n >> 0$
- if length$(f^n(\gamma^c)) \to \infty$ then area$(W^u(f^n(\gamma^c))) \to \infty$
- (bounded angles)
- $\to$ CONTRADICTION
- $\Rightarrow f^n(\gamma^c)$ bounded

step 2

- $\gamma^c \subset T$ small
- $\varepsilon > 0$ small
- $\Rightarrow W^u_\varepsilon(f^n(\gamma^c)) \subset f^n(W^u_\varepsilon(\gamma^c))$ for $n >> 0$
- if length$(f^n(\gamma^c)) \to \infty$ then area$(W^u(f^n(\gamma^c))) \to \infty$
- (bounded angles)
- $\to$ CONTRADICTION
- $\Rightarrow f^n(\gamma^c)$ bounded
- $\Rightarrow h(\gamma^c) \subset W^s(x)$

step 2

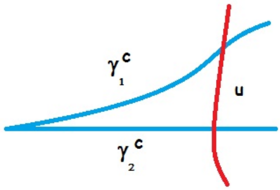
proposition

E^c is uniquely integrable on T

step 2

proposition

E^c is uniquely integrable on T



step 2

conclusion

$h|_T$ takes

step 2

conclusion

$h|_T$ takes

1. W_f^u into W_A^u

step 2

conclusion

$h|_T$ takes

1. W_f^u into W_A^u
2. W_f^c into W_A^s

step 3

step 3

There exists a closed loop transverse to $\tilde{\mathcal{F}}^{cu}$ in $\tilde{M}$

step 3

- $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$

step 3

- $\sup_{y \in T} \mathrm{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc

step 3

- $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc
- Let $p \in T \times 0$ periodic point

step 3

- $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc
- Let $p \in T \times 0$ periodic point
- such that $h^{-1}(h(p)) \cap (T \times 0)$ small center arc

step 3

- $\sup_{y \in T} \mathrm{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc
- Let $p \in T \times 0$ periodic point
- such that $h^{-1}(h(p)) \cap (T \times 0)$ small center arc
- U small neighborhood of $h^{-1}(h(p)) \cap (T \times 0)$ in $T \times I$

step 3

- $\sup_{y\in T}\text{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y)\cap(T\times 0)\subset W^c(y)$ connected arc
- Let $p\in T\times 0$ periodic point
- such that $h^{-1}(h(p))\cap(T\times 0)$ small center arc
- U small neighborhood of $h^{-1}(h(p))\cap(T\times 0)$ in $T\times I$

!

$h^{-1}(h(p))\cap U\subset W^{cs}_{loc}(p)$

step 3

- $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc
- Let $p \in T \times 0$ periodic point
- such that $h^{-1}(h(p)) \cap (T \times 0)$ small center arc
- U small neighborhood of $h^{-1}(h(p)) \cap (T \times 0)$ in $T \times I$

!

$h^{-1}(h(p)) \cap U \subset W^{cs}_{loc}(p)$

- let $y \in h^{-1}(h(p)) \cap U$

step 3

- $\sup_{y \in T} \mathrm{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc
- Let $p \in T \times 0$ periodic point
- such that $h^{-1}(h(p)) \cap (T \times 0)$ small center arc
- U small neighborhood of $h^{-1}(h(p)) \cap (T \times 0)$ in $T \times I$

!

$h^{-1}(h(p)) \cap U \subset W^{cs}_{loc}(p)$

- let $y \in h^{-1}(h(p)) \cap U$
- let $z \in W^s_{loc}(y) \cap T \times 0$

step 3

- $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$
- $h^{-1}(y) \cap (T \times 0) \subset W^c(y)$ connected arc
- Let $p \in T \times 0$ periodic point
- such that $h^{-1}(h(p)) \cap (T \times 0)$ small center arc
- U small neighborhood of $h^{-1}(h(p)) \cap (T \times 0)$ in $T \times I$

!

$h^{-1}(h(p)) \cap U \subset W^{cs}_{loc}(p)$

- let $y \in h^{-1}(h(p)) \cap U$
- let $z \in W^s_{loc}(y) \cap T \times 0$
- $\Rightarrow h(z) \in h(W^s_{loc,f}(y)) \subset W^s_{loc,A}(h(p))$

step 3

- $W^{cs}_{loc}(p)$ foliated by center arcs (just $\cap\mathcal{F}^{cu}$)

step 3

- $W^{cs}_{loc}(p)$ foliated by center arcs (just $\cap \mathcal{F}^{cu}$)
- each center arc $\cap h^{-1}(h(p))$

step 3

- $W^{cs}_{loc}(p)$ foliated by center arcs (just $\cap\mathcal{F}^{cu}$)
- each center arc $\cap h^{-1}(h(p))$
- the same happens in the universal cover

step 3

- $W^{cs}_{loc}(p)$ foliated by center arcs (just $\cap \mathcal{F}^{cu}$)
- each center arc $\cap h^{-1}(h(p))$
- the same happens in the universal cover
- remember $\sup_{y \in T} \text{diam}(\bar{h}^{-1}(y)) < C$

step 3

- $W^{cs}_{loc}(p)$ foliated by center arcs (just $\cap \mathcal{F}^{cu}$)
- each center arc $\cap h^{-1}(h(p))$
- the same happens in the universal cover
- remember $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$
- take $\varepsilon > 0$ small and $N > 0$ such that

step 3

- $W^{cs}_{loc}(p)$ foliated by center arcs (just $\cap \mathcal{F}^{cu}$)
- each center arc $\cap h^{-1}(h(p))$
- the same happens in the universal cover
- remember $\sup_{y \in T} \operatorname{diam}(\bar{h}^{-1}(y)) < C$
- take $\varepsilon > 0$ small and $N > 0$ such that
- $\operatorname{diam} X < C \Rightarrow N(X, \varepsilon) < N$

step 3

- Take $\{x_1, \ldots, x_N\} \subset h^{-1}(h(p)) \cap W^{cs}_{loc}(\bar{p})$

step 3

- Take $\{x_1, \dots, x_N\} \subset h^{-1}(h(p)) \cap W^{cs}_{loc}(\bar{p})$
- in different center curves

step 3

- Take $\{x_1, \ldots, x_N\} \subset h^{-1}(h(p)) \cap W^{cs}_{loc}(\bar{p})$
- in different center curves
- $\Rightarrow \operatorname{diam}\{f^n(x_1), \ldots, f^n(x_N)\} < C$ for all $n \in \mathbb{Z}$

step 3

- Take $\{x_1, \ldots, x_N\} \subset h^{-1}(h(p)) \cap W^{cs}_{loc}(\bar{p})$
- in different center curves
- $\Rightarrow$ diam$\{f^n(x_1), \ldots, f^n(x_N)\} < C$ for all $n \in \mathbb{Z}$
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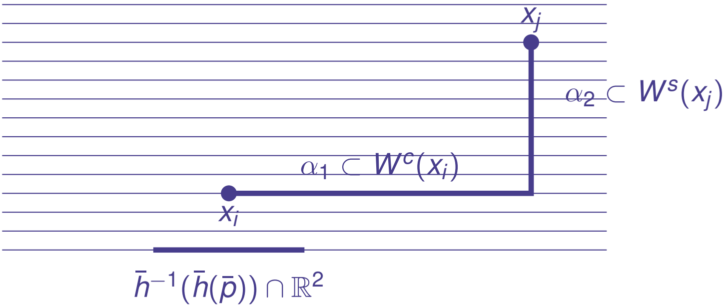
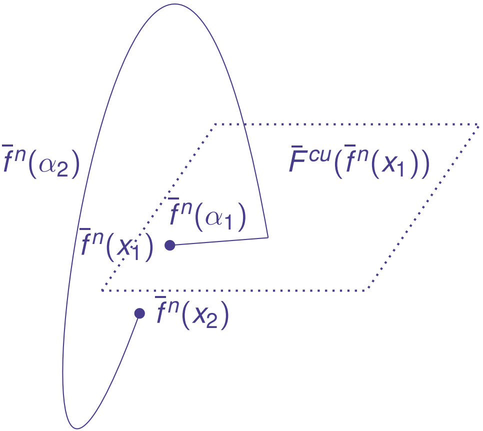
- 

Figure: $W^{cs}(\bar{p})$

step 3

when $n_k \to \infty$



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