

## FOCUS PROGRAM

# Nonlinear Dispersive Partial Differential Equations and Inverse Scattering

**From July 31 to August 23, 2017, the Fields Institute Focus Program on Nonlinear Dispersive Partial Differential Equations and Inverse scattering brought together specialists in completely integrable systems, inverse scattering, and partial differential equations.**

The centerpiece of the program was the series of three Coxeter lectures delivered by Percy Deift. Deift began his lectures by describing the structure underlying the integrability of the defocusing nonlinear Schrödinger equation, and after a tour through other topics ranging from Hamiltonian systems to random matrix theory, he concluded his lectures by proposing a working definition of what it means for a problem to be integrable. The lectures drew a large audience including researchers in nearby areas, graduate students, and researchers in tangentially-related areas.

The Focus Program began with one-week a summer school for graduate students and early career researchers which developed important background material and described the landscape of integrable systems techniques in dispersive partial differential equations. Some lecturers also emphasized new phenomena and applications. Examples include the lectures of Walter Craig on interacting vortices and the presentation of Patrick Gérard on the integrability of the cubic Szegő equation and the growth of high Sobolev norms in its solutions. One researcher wrote afterwards:

“I found it to be a very beneficial meeting. My grad students were also very happy with the summer school. Several times now I have started explaining something to them, but they tell

me they already know how to do it from the summer school.”

During the following two weeks, invited lecturers painted a compelling picture of current research on completely integrable and PDE techniques in dispersive nonlinear waves. Highlights of the first workshop week included lectures by Daniel Tataru and Rowan Killip on derived conserved quantities for the KdV and NLS equations to initial data in rough Sobolev spaces, dramatically increasing the reach of completely integrable methods. In the second week, Adrian Nachman announced a dramatic and far-reaching extension of the solution of the Cauchy problem for the defocusing Davey-Stewartson II equation, joint with his student Idan Regev and Daniel Tataru. Their work combines inverse scattering techniques with deep results in harmonic analysis including new fractional integral estimates and new  $L^2$ -boundedness theorems for pseudo differential operators. Both these results and many others presented at the conference underscore the timeliness of its subject and the importance of bringing researchers in these communities together.



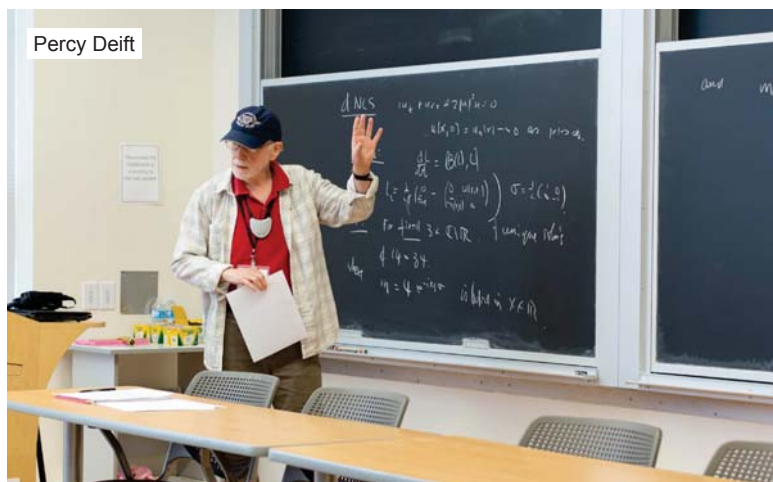
Participants of the Summer School

Finally, the poster presentations allowed graduate students and junior researchers to discuss their work with others and gather useful feedback. As an example, postdoc Deniz Bilman presented a poster on a new approach to the initial-value problem for the focusing nonlinear Schrödinger equation that drew many comments and allowed Bilman to learn first-hand from others working in the field, such as Gino Biondini, Percy Deift, Ken McLaughlin, and Barbara Prinari.

A number of participants initiated new collaborations or advanced international collaborations already in progress.

- Olga Assainova (graduate student), Christian Klein, Ken McLaughlin, and Peter Miller made progress that led to a paper analyzing the reflection coefficient for the defocusing Davey-Stewartson-II equation in the semiclassical limit. This paper was submitted just recently for publication.
- A group including Joel Klipfel (graduate student), Peter Perry, Allen Wu, and Peter Miller made progress in initiating a study of the inverse-scattering transform for the intermediate long-wave equation.
- Robert Buckingham, Robert Jenkins, and Peter Miller made progress in understanding a family of three-sheeted genus-zero Riemann surfaces underlying certain exact solutions of the three-wave resonant interaction equations. These solutions appear naturally in an asymptotic context during the interaction of monochromatic waves.
- David Smith and Deniz Bilman made progress on a collaborative project on interface problems for the nonlinear Schrödinger equation, with Deniz Bilman, and a project on the Massive Thirring Model with Robert Buckingham.
- Mark Hoefer and Michelle Maiden (graduate student) have begun a collaboration with Gino Biondini, initiated at Fields and due to interactions there. ❖

— Peter Miller, Peter Perry, Jean-Claude Saut, Catherine Sulem



## Spotlight



THE FIELDS Research Fellowship program was launched in 2017 as a way to bring high-calibre mathematicians to the Institute for a period of full-time “research in residence”. Professor Carlos Tomei was one of the first Fields Research Fellows, staying at the Institute from July 2 to August 10, 2017 to work with Professor Peter Gibson from York University.

Tomei is a Full Professor in the Department of Mathematics at the Pontifical Catholic University of Rio de Janeiro in Brazil. His mathematical interests lie in analysis, with frequent interlacing of theoretical and computational aspects. He has been working with Gibson intermittently for ten years, and it was finally time for Tomei to

reciprocate Gibson’s visits to Rio.

“The visit to the Fields was such a wonderful opportunity. Everything was motivating: meeting old friends and making new ones, hours alternating walks and blackboard sessions.”

As an extra bonus, the last three weeks of Tomei’s stay coincided with a series of events related to subjects very close to his heart, culminating in the Coxeter Lectures given by Percy Deift, Tomei’s former PhD supervisor and a constant inspiration.

“The six weeks we spent together at the Fields were very fruitful: we made progress on all fronts and Peter [Gibson] was the perfect host.” ❖

— Malgosia Ip