Global dominated splitting or zero Lyapunov exponent almost everywhere

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This talk is based on joint work with Artur Avila.

Let M be a compact manifold of dimension at least 2, and let m be a normalized volume measure. Let $\mathrm{Diff}^1_m(M)$ be the set of C^1 conservative (i.e. m-preserving) diffeomorphisms, endowed with the C^1 topology.

It was shown in [BV] that *for a generic f in $\mathrm{Diff}^1_m(M)$, the Oseledets splitting along m-almost every orbit is either trivial or dominated.* In particular, the manifold M equals $Z \sqcup \bigcup_n \Lambda_n$ mod 0, where Z is the set where all Lyapunov exponents are zero, and each Λ_n is a f-invariant Borel set where the Oseledets splitting is non-trivial and (uniformly) dominated.

This suggests the following question: *Is it true that generically, either all exponents are zero almost everywhere or there is a global dominated splitting? In other words, is it true that either Z or Λ_1 has full measure?* The answer is yes if M is 2-dimensional, by [B].

We answer a weaker version of the question above, proving that *generically either at almost every point at least one Lyapunov exponent vanishes, or there is a global dominated splitting $TM = E^{cu} \oplus E^{cs}$.* In fact, in the second alternative, we prove more:

1. the set $\mathrm{NUH}(f)$ formed by the Lyapunov regular points with no zero exponents is essentially dense in M (that is, its intersection with every nonempty open set has positive m-measure).
2. the measure $m|\mathrm{NUH}(f)$ is ergodic for f;
3. m-almost every point $x \in \mathrm{NUH}(f)$ has index equal to $\dim E^{cu}$;

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In the proof we use some recent technology, namely, the ergodicity criterion of [RRTU], the denseness of C^2 maps in $\mathrm{Diff}^1_m(M)$ [Av], and the C^1 dominated version of Pesin theory [ABC].

We also used some new generic properties that are of independent interest, as the *generic continuity of the ergodic decomposition.*

References

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