“non-hiperbolicities”

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PUC-Rio

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thanks:

to the Brazil-France Cooperation in Mathematics,

for the long-lasting support....

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- What structures cannot exist in the hyperbolic case but must be present in its complement?

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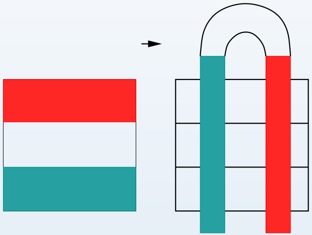

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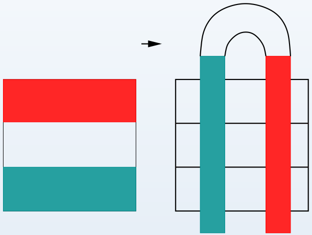

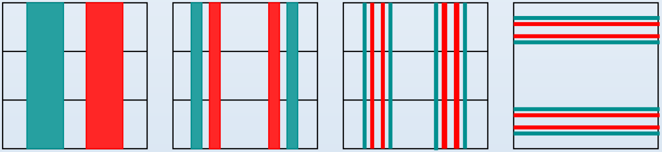

symbolic dynamics (i)

Each orbit in the horseshoe Λ is represented by a sequence of **0** (iterate in the red rectangle) and **1** (iterate in the blue rectangle):

... **1 0 1 0 1 1 0**....

symbolic dynamics (conjugation to shifts)

$$\Sigma = \{0,1\}^{\mathbb{Z}}, \quad \text{with some metric...}$$

$$\sigma : \Sigma \to \Sigma, \qquad (x_i) \mapsto (y_i), \quad y_i = x_{i+1}.$$

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commutative diagramm: h is a homeomorphism (conjugation).

$$\begin{array}{ccc} \Sigma & \longrightarrow \sigma \longrightarrow & \Sigma \\ h \downarrow & & \downarrow h \\ \Lambda & \longrightarrow f \longrightarrow & \Lambda \end{array}$$

translate shift properties (symbolic dynamics) to the ambient dynamics:

- mixing, transitivity (dense orbits, recurrences....),
- infinitely many periodic points.

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directions corresponding to uniform contraction (stable) and expansion (unstable).

$f\colon M \to M$, diffeo., M compact and closed,

Λ: f-invariant ($f(\Lambda) = \Lambda$) compact set.

hyperbolic set:

$T_\Lambda M = E^s \oplus E^u$,

Df-invariant and constants $C > 0$ and $\lambda < 1$ with

$$|Df^m(v^s)| \le C\,\lambda^m\,|v^s|, \quad |Df^{-m}(v^u)| \le C\,\lambda^m\,|v^u|,$$

for every $m \ge 0$, $v^s \in E^s_x$ and $v^u \in E^u_x$, and all $x \in \Lambda$.

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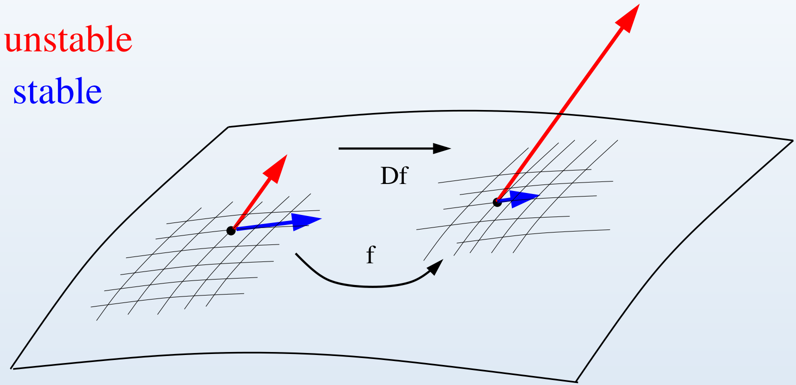

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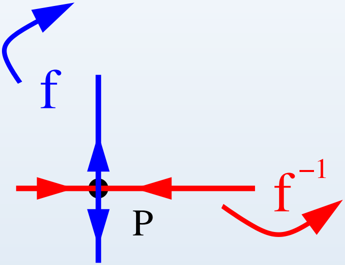

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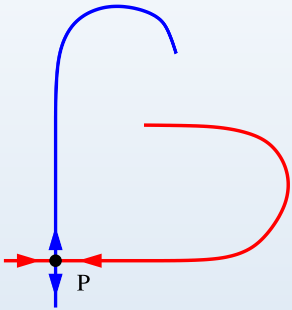

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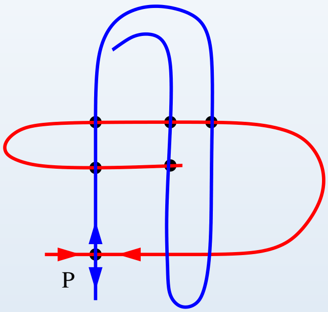

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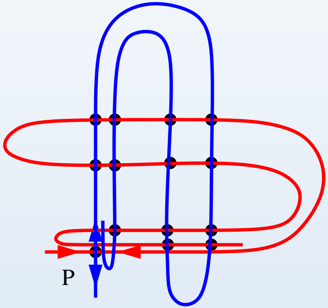

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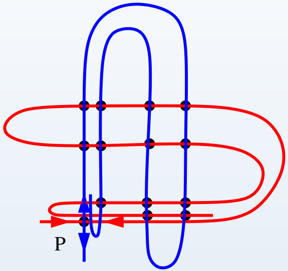

H(*P*) homoclinic class of *P*: closure of the transverse intersections its invariant (stable and unstable) manifolds.

- mixing, transitivity (dense orbits, recurrences....),
- infinitely many periodic points,
- in some cases *H*(*P*) fails to be hyperbolic...

hyperbolic summary

general facts

- There is a complete theory of hyperbolic systems: geometric, topological, and ergodic (probabilistic) aspects.
- Nonhyperbolic systems are quite frequent and many of them exhibit "some (weak) hiperbolicity"
- non-hyperbolicities: non-uniform, partial, singular, dominated splittings....
- A little hyperbolicity goes a long way (Pugh-Shub).

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Palis conjecture

Dichotomy: hyperbolicity versus cycles

cycles:

- homoclinic tangencies (dim ≥ 2),
- heterodimensional cycles (dim ≥ 3).

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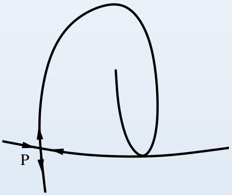

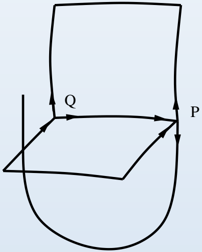

some results

the conjecture holds for

- circle maps (Peixoto),
- C^1 surface diffeomorphisms (Pujals-Sambarino),
- C^1 tame diffeomorphisms (Bonatti-D.).

tame diffeomorphisms

Those having stably finitely many homoclinic classes.

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typical non-hyperbolicities

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- uniform rate of expansion and contraction,
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two types of non-hyperbolicty:

- critical,
- non-critical.

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- models: Quadratic family, Hénon-like maps,
- the rate expansion is not uniform, the angles between the invariant directions are not uniform,
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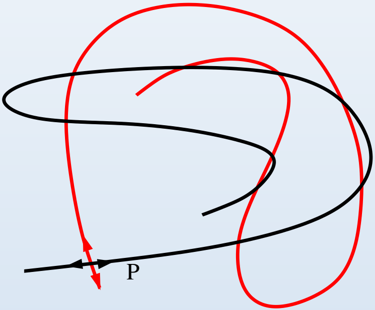

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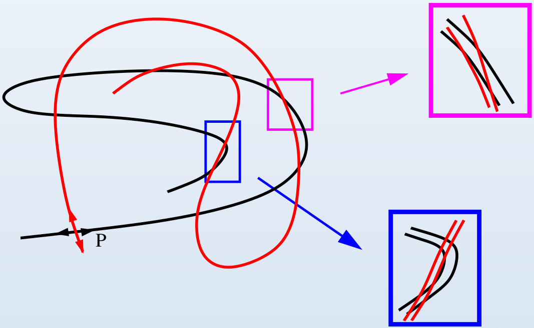

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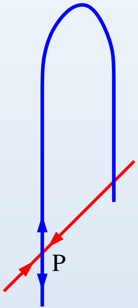

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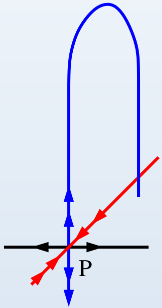

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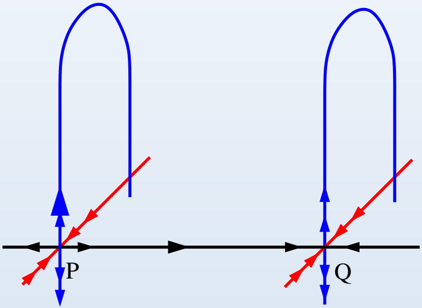

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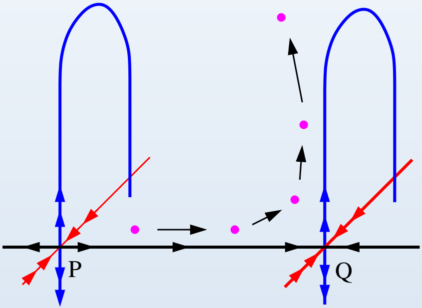

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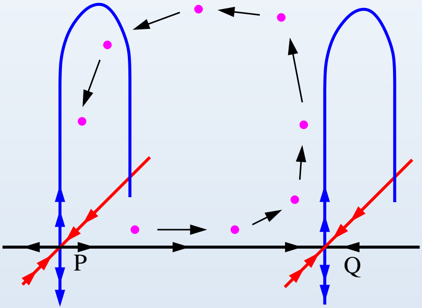

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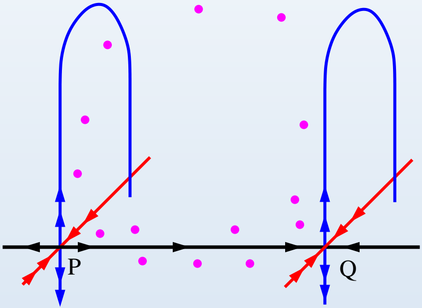
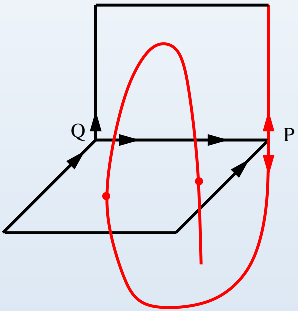


non-hyperbolic homoclinic classes

$H(P)$ may content saddles of index (dimension of stable bundle) different from the one of P. Typical non-dynamical feature.



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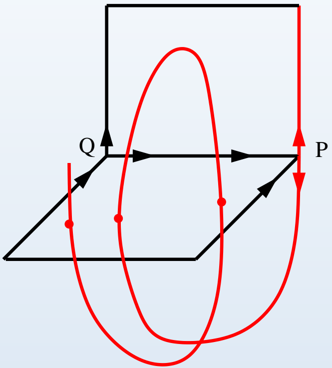

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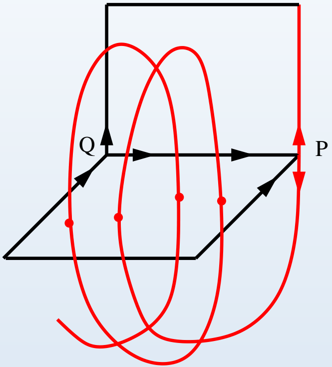

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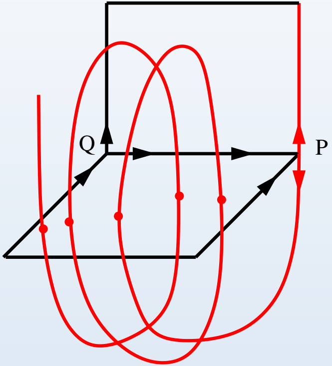

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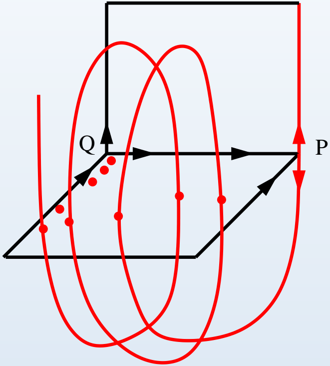

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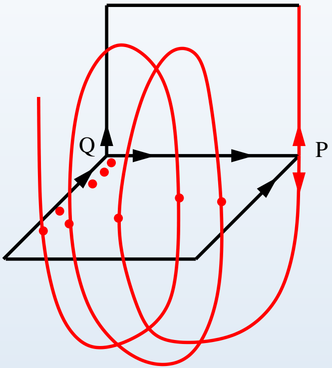

caution: a homoclinic class whose saddles have all the same index may be non-hyperbolic....

non-critical model: skew products

f_0, f_1 circle maps,

f_0 East-West map, f_1 irrational rotation.

$\sigma : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ horseshoe (shift map).

skew-product (partially hyperbolic map)

$$F : \{0,1\}^{\mathbb{Z}} \times \mathbb{S}^1 \to \{0,1\}^{\mathbb{Z}} \times \mathbb{S}^1, \quad F(\alpha, x) = (\sigma(\alpha), f_{\alpha_0}(x)).$$

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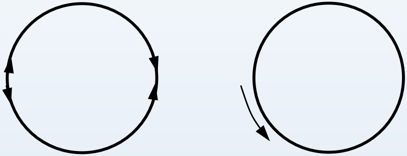

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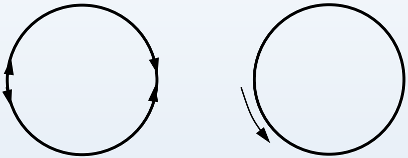

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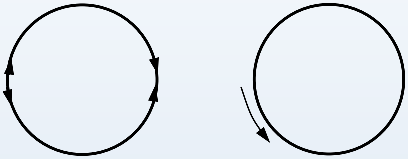

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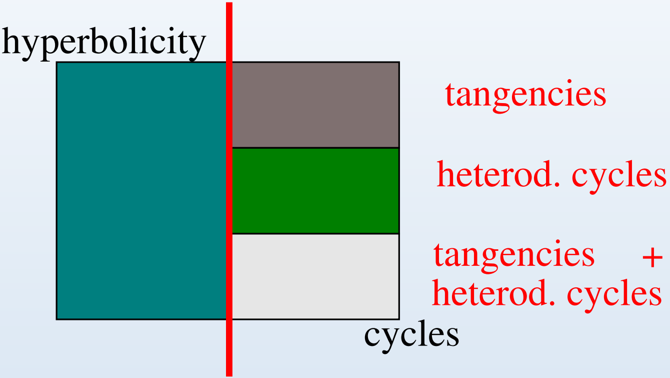

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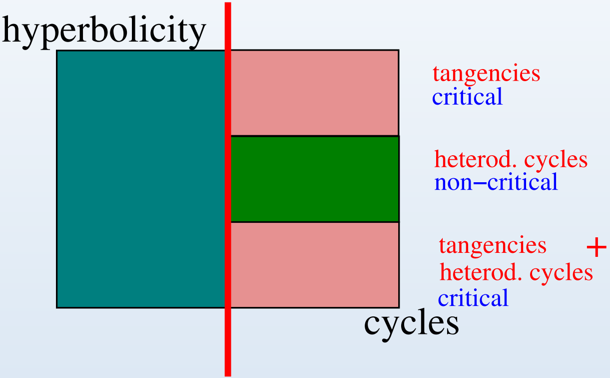

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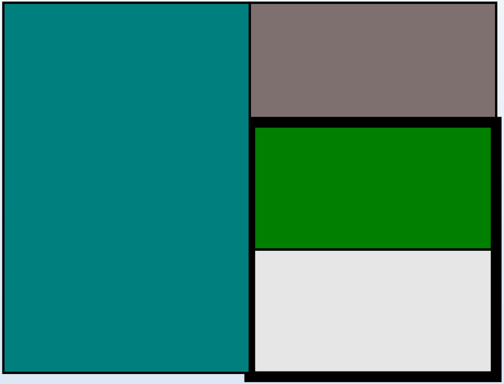

robust het. cycles

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robust tangencies

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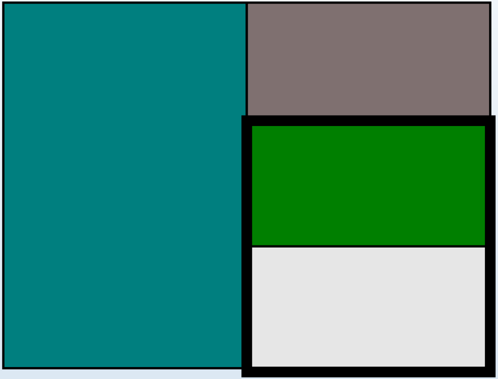

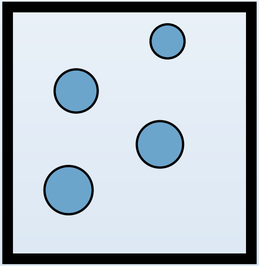

heterodimensional cycles associated to hyperbolic sets

Λ and Σ hyperbolic sets, different indices

$$W^s(\Lambda) \cap W^u(\Sigma) \neq \emptyset \quad W^u(\Lambda) \cap W^s(\Sigma) \neq \emptyset.$$

similarly for homoclinic tangencies.

robust cycles (heterodim. cycles and tangencies)

every g close to f has a cycle.

Kupka-Smale Theorem

generically, periodic points are hyperbolic and their invariant manifolds are in general position (transversality).

thus: robust cycles involve some non-trivial hyperbolic set.

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non-hyperbolic features

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- bifurcations of periodic orbits (saddle-node, flip, Hopf),
- absence of shadowing properties,
- cycles,
- Newhouse-like phenomena: super-exponential growth of the number of periodic points,
- non-hyperbolic ergodic measures with large support,
- non-existence of symbolic extensions.

question

Which dynamical features are typical of each form of non-hyperbolicity?

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diffeo $f : M \to M$, $\dim M = n$,

μ ergodic measure of f: $\mu(f^{-1}(A)) = A$ implies $\mu(A) = 0, 1$,

there are Λ of full μ-measure,

$\chi_\mu^1 \leq \chi_\mu^2 \leq \cdots \leq \chi^n$

for all $x \in \Lambda$ and all $v \in T_x M$, $v \neq 0$,

$$\lim_{n\to\infty} \frac{1}{n} \log ||Df^n(v)|| = \chi_\mu^i, \quad \text{some } i = 1, \ldots, n.$$

χ_μ^i is the i-th Lyapunov exponent of μ.

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symbolic extensions

$f: X \to X$, homeomorphism,

(X, f) has a symbolic extension if there exists a subshift (finitely many symbols) (Y, σ) and a continuous surjective map $\pi : Y \to X$ such that

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(Y, σ) is called an extension of (X, f)

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remark (!!): super-exponential growth is compatible with the existence of symbolic extensions.

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$f\colon M \to M$, diffeo., M compact and closed,

Λ: f-invariant ($f(\Lambda) = \Lambda$) compact set.

dominated splitting: $T_\Lambda M = E \oplus F$, Df-invariant and there is m with

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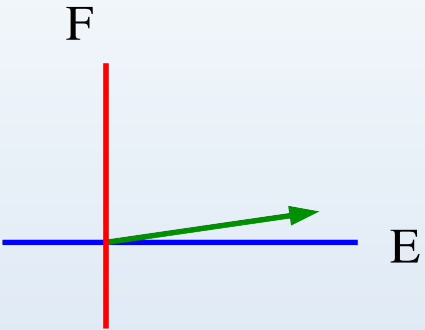

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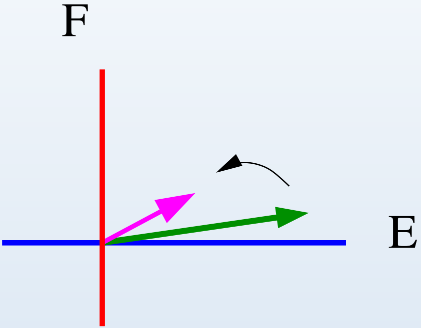

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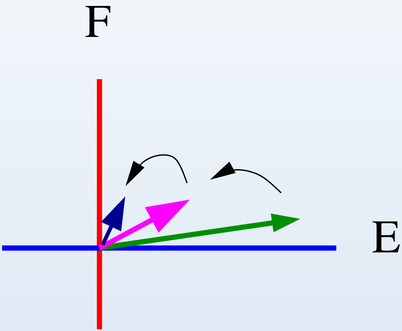

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for simplicity: dimension of M is three

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- non-critical case: three bundles $E^s \oplus E^c \oplus E^u$, E^s and E^u hyperbolic.
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non-hyperbolic features

C^1 generic setting (homoclinic classes with saddles of different indices):

non-hyperbolicity always implies:

- supergrowth of the the number of periodic points (Bonatti-D.-Fisher),
- no-shadowing property (Sakai, Yorke-Yuan, Abdenur-D., Bonatti-D-Turcat),
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further information, non-critical case

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summarizing table

dimension three:

dynamics/splitting	3 bundles	2 bundles	non-exist.
• Robust tangencies	No	Yes	Yes
• Robust het. cycles	Yes	Yes	Yes
• non-hyperbolic measures	Yes	Yes	Yes
- full support	Yes	Yes?	Yes?
- # zero exponents	1	2?	3?
• symbolic extensions	Yes	No	No
• ∞-sinks/sources	No	depend	Yes