“non-hyperbolicities”

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PUC-Rio

III CLAM, Santiago, September 2009

beyond hyperbolicity

goals, questions

How to characterize the absence of (uniform) hyperbolicity?

What structures cannot exist in the hyperbolic case but must be present in its complement?

general facts

Many nonhyperbolic systems exhibit "some (weak) hiperbolicity."

non-uniform, partial, singular, dominated splittings....

A little hyperbolicity goes a long way (Pugh-Shub).

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Palis conjecture

Dichotomy: hyperbolicity versus cycles

cycles:

- homoclinic tangencies (dim ≥ 2),
- heterodimensional cycles (dim ≥ 3).

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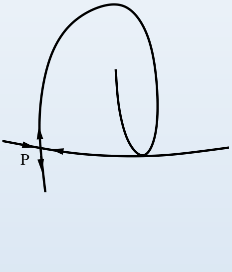

some results

the conjecture holds for

- circle maps (Peixoto),
- C^1 surface diffeomorphisms (Pujals-Sambarino)
- C^1 tame diffeomorphisms $n \geq 3$ (Bonatti-D.).

tame diffeomorphisms

Those having stably finitely many homoclinic classes.

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Heterodimensional cycles associated to hyperbolic sets

Λ and Σ hyperbolic basic sets, different indices

$$W^s(\Lambda) \cap W^u(\Sigma) \neq \emptyset \quad W^u(\Lambda) \cap W^s(\Sigma) \neq \emptyset.$$

similarly for homoclinic tangencies.

robust cycles (heterodim. cycles and tangencies)

every g close to f has a cycle.

Kupka-Smale Theorem

generically, periodic points are hyperbolic and their invariant manifolds are in general position (transversality).

thus: robust cycles involve some non-trivial hyperbolic set.

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non-hyperbolic features

some non-hyperbolic features

- bifurcations of periodic orbits (saddle-node, flip, Hopf),
- absence of shadowing properties,
- cycles,
- Newhouse-like phenomena: super-exponential growth of the number of periodic points,
- non-hyperbolic ergodic measures with large support,
- non-existence of symbolic extensions.

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diffeo $f : M \to M$, $\dim M = n$,

μ ergodic measure of f: $\mu(f^{-1}(A)) = A$ implies $\mu(A) = 0, 1$,

there are Λ of full μ-measure,

$\chi^1_\mu \le \chi^2_\mu \le \cdots \le \chi^n$

for all $x \in \Lambda$ and all $v \in T_x M$, $v \neq 0$,

$$\lim_{n\to\infty} \frac{1}{n} \log ||Df^n(v)|| = \chi^i_\mu, \quad \text{some } i = 1, \ldots, n.$$

χ^i_μ is the i-th Lyapunov exponent of μ.

μ is non-hyperbolic if $\chi^i_\mu = 0$ for some i.

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f is Artin-Mazur (A-M) if the number $\mathbf{P}_n(f)$ of isolated periodic points of period n of f satisfies:

$$\mathbf{P}_k(f) \leq \exp(Ck).$$

A-M maps are dense in the space of C^r-maps but no C^r-generic, $r \geq 1$ (Kaloshin).

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$$\limsup \frac{\mathbf{P}_k}{a_k} = \infty, \quad \text{if } f \in R(a).$$

super-exponential growth is common in non-hyperbolic dynamics (Kaloshin for C^2-tangencies and Bonatti-D.-Fisher for C^1-non-hyperbolic dynamics).

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symbolic extensions

$f\colon X \to X$, homeomorphism,

(X, f) has a symbolic extension if there exists a subshift (finitely many symbols) (Σ, σ) and a continuous surjective map $\pi : \Sigma \to X$ such that

$$\pi \circ \sigma = f \circ \pi.$$

(Σ, σ) is called an extension of (X, f)

(X, f) is a factor of (Σ, σ).

remark (!!): super-exponential growth is compatible with the existence of symbolic extensions, periodic orbits may be covered by non-periodic ones.

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C^1-diffeomorphisms,

context: C^1-generic dynamics (i.e., study of (locally) residual subsets)

consequences: generically, periodic points are hyperbolic and their invariant manifolds are in general positions (transversality)

thus: local bifurcations of periodic points and cycles associated to saddles can be discarded (?!).

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We are interested in these bifurcations and (mostly) in their dynamica consequences.... typical dynamical features generated.

Reformulation: we replace cycles associated to saddles by cycles associated to hyperbolic sets.

Facts: local bifurcations of periodic points and cycles yield cycles associated to sets, these cycles can be robust.

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Λ: f-invariant ($f(\Lambda) = \Lambda$) compact set.

dominated splitting: $T_\Lambda M = E \oplus F$, Df-invariant and there is m with

$$\frac{|Df^m(v)|}{|(Df^m(w))|} \le \frac{1}{2}$$

for every unitary vectors $v \in E_x$ and $w \in F_x$ and all $x \in \Lambda$.

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dominated splitting: $T_\Lambda M = E \oplus F$, Df-invariant and there is m with

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for every unitary vectors $v \in E_x$ and $w \in F_x$ and all $x \in \Lambda$.

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$\Lambda = H(p, f)$, p a saddle and $H(p, f)$ closure of transverse intersections of the invariant manifolds of p.

$H(p, f)$ may content saddles of index (dimension of stable bundle) different from the one of p. Typical non-dynamical feature.

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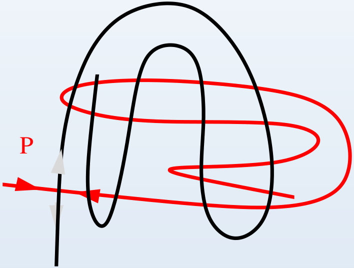

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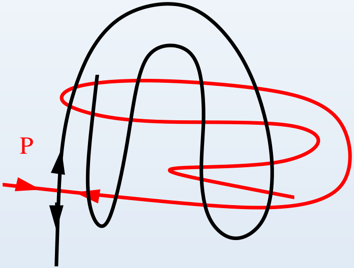

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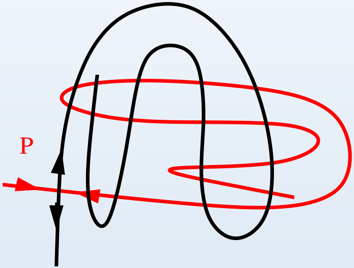

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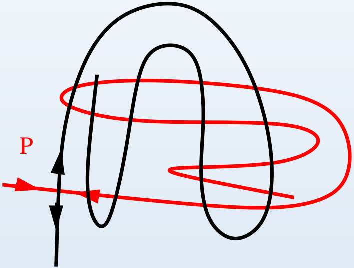

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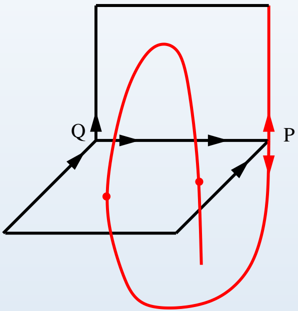

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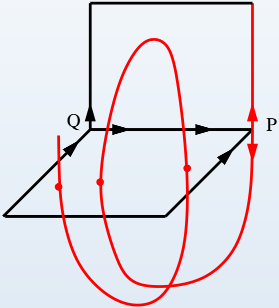

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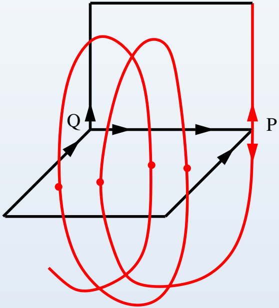

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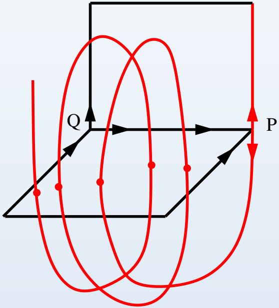

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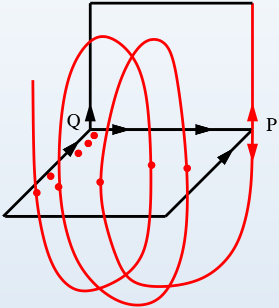

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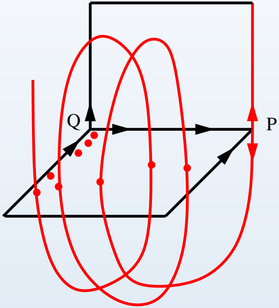

caution: a homoclinic class whose saddles have all the same index may be non-hyperbolic....

types of splittings

for simplicity: dimension of M is three

possibilities for the finest dominated splitting of a homoclinic class.

- non-critical case: three bundles $E^s \oplus E^c \oplus E^u$, E^s and E^u hyperbolic.
- critical case:
  - two bundles: $E^s \oplus E^{cu}$ or $E^{cs} \oplus E^u$, E^s and E^u one-dimensional and hyperbolic.
  - non-existence.

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dynamical features and types of splitting

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non-hyperbolicity (homoclinic classes with saddles of different indices) always implies:

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summarizing table

dimension three:

dynamics/splitting	3 bundles	2 bundles	non-exist.
• Robust tangencies	No	Yes	Yes
• Robust het. cycles	Yes	Yes	Yes
• non-hyperbolic measures	Yes	Yes	Yes
- full support	Yes	Yes?	Yes?
- # zero exponents	1	2?	3?
• symbolic extensions	Yes	No	No
• ∞-sinks/sources	No	depend	Yes