SPACE-TIME STRUCTURE AND QUANTUM MECHANICS

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Abstract

Various studies suggest a close relation between space-time structure and the general characteristics of quantum mechanics. The final explanation for these, such as linearity, would be forthcoming only through quantum gravity.

1 . INTRODUCTION

There are two universal features of modern day physics. All phenomena take place in space-time and all phenomena are (in principle) subject to quantum mechanics. Two universalities seem too much for anyone believing in the unity of the physical world and one is led to inquire if these two aspects are not just two facets of the same underlying physical reality. We have come to believe that the following correspondence exists between the two major structures:

Space-Time	Quantum Mechanics
Lorentz signature	Linearity and Hilbert space
Simultaneity is indeterminate	Outcomes are indeterminate
Four dimensional	Complex amplitudes

Here we shall only concentrate on the first of the above correspondences as the evidence for the others is still very sketchy. Much of the evidence for the first correspondence comes from attempts to formulate a non-linear version of quantum mechanics and the seemingly insurmountable difficulties with relativity that this entails. In the final analysis it appears that a full understanding of the main features of quantum theory can only be achieved through a joint formulation with space-time. The two must arise together.

2. MUST QUANTUM MECHANICS BE LINEAR?

Linearity is certainly a striking feature of quantum mechanics, but what is it due to? We believe that the main assumptions that would lead to linearity are the following: 1) Space-time is Lorentzian. 2) Existence of self-subsisting physical states. These are characterized by the fact that evolution is independent of creation process and subsequent measurement and that they undergo "collapse" in measurements 3) Equivalence of time-like and space-like conditioning.

Some explanation is in order. What do we mean by "self-subsisting physical states"? In quantum theory, a physical state evolves deterministically by a unitary group. The state is generally created at some time and destroyed later in a measurement (or something akin to it), but the deterministic evolution can be extended to both temporal infinities. The state is thus an autonomous physical entity having no memory of its birth nor any prescience of its demise. Regardless of the ontological status of such entities, physical theories use them as formal devices to compute joint probabilities of observed events. A sequence of events is then seen as the interaction of a state with a measurement apparatus by which the state is modified and then evolves until the next interaction when it suffers another modification followed by another evolution, and so on. Thus joint probabilities of events are computed using the interpolating existence of evolving self-subsisting entities. This is not logically necessary. One can maintain that physical states are not really necessary to do physics, as one can conceive of ways of calculating joint probabilities without their use. In fact certain patterns of probabilities cannot be interpreted this way [1]. The "consistent histories" approach to quantum mechanics [2, 3] in fact abolishes to a large extent self-subsisting physical states and can easily produce examples [2] where joint probabilities cannot be explained by them.

To explain the third assumption, consider the following simplified situation. Suppose one wants to study right-hand circularly polarized photons. Well, put an appropriate filter in front of a light source and observe at will. Alternatively, set up an EPR-type arrangement that creates singlet two-photon states with the individual photons flying off in opposite directions. Put the same filter on the *distant* arm of apparatus and *nothing* on the near arm. Observe at will. Half of the photons observed are right-hand circularly polarized and half are in the orthogonal left-hand circularly polarized state, and there is no way of knowing which is which while measurements are being made. This poses no problem, just wait until the results (passage through the filter or not) at the distant arm of each photon pair are available and simply throw out all the experimental data for the instances where the distant photon did not pass through the filter. Now one has data of just the right-hand circularly polarized photons at the near arm. In the first kind of experiment we calculate probabilities conditioned to an event (passage through a filter) in the causal past of the relevant measurement, in the second, the probabilities are conditioned to an event space-like to the relevant measurement, and data analysis is done *a posteriori*. The fact that these two experimental procedures are equivalent is a feature of ordinary quantum mechanics and depends on the existence of a particular entangled state with long-range correlations, the two-photon singlet in the our example.

Arguments brought forth in [4, 5] lead to the suggestion that the above assumptions are the main characteristics behind a Hilbert-space model for quantum mechanics. From this point on, one has various arguments [6, 7, 8] that lead to linearity.

There are relativistic constraint on joint probabilities of experimental outcomes. Consider a measurement with space-like separated instrumental events such as a correlation measurement of the EPR type. In one frame the measurements on the two parts are simultaneous and so can be considered as just parts of a single measurement, while in another frame the two measurements are successive with intervening time evolution. These two description must be equivalent and produce the same observable results, which translate to constraints imposed by relativity and that relate the measurement process to the evolution. What is remarkable is that these same constraints imposed on generally *compatible* instruments (and not just on space-like separated ones) lead in several axiomatic schemes [9, 10, 11, 12, 13] to a Hilbert-space model for physical propositions, for one can then deduce the "covering law" of Piron's [14] axiomatic approach to quantum mechanics. The base field (complex numbers) of the Hilbert space is not determined, but we feel this has a different genesis. Thus there seems to be a relation between space-time structure (relativistic causality in particular) and the Hilbert-space model of quantum mechanics [5, 13]. Thus to arrive at a Hilbert-space model one needs somehow to generalize to compatible *time-like* measurements a condition deduced for *space-like* measurements. The equivalence principle allude to before allows for this. If one assumes, as is the case for ordinary quantum mechanics, that to any time-like experimental arrangement with compatible instruments, there is an equivalent space-like arrangement performed on an appropriate long-range correlated states, then one completes the argument toward the covering law and a Hilbert-space model of quantum mechanics.

With these results it seem very doubtful that a causal relativistic nonlinear quantum mechanics is possible at all. A notable example is Weinberg's theory and he writes [15] "I could not find any way to extend the nonlinear version of quantum mechanics to theories based on Einstein's special theory of relativity", thereby abandoning his non-linear theory altogether.

On the basis of items (1-3) one can understand the failure. Self-subsisting physical states is such an ingrained notion in physics that it is naturally included as a basic ingredient, so item (2) is almost automatic. Item (3)) in a nonrelativistic theory is not even expressed, and whatever notion of compatibility of measurements is used, this item would more than likely be automatically incorporated into any attempt at a relativistic theory. Item (1) is what one is trying to achieve. But now we have a contradictory mixture as these items lead to a standard Hilbert-space model which precludes nonlinearities.

Arguments against nonlinear theories have been brought forth by various authors [4, 16, 17, 18] and are all based on the observation that instantaneous wave-function collapse along with non-linearity allows one to send instantaneous signals. The problem is in fact more insidious and the proposed nonlinear theories have a more fundamental incompatibility with relativity [19].

On the other hand, non-linear theories, besides arising from basic speculation of the "why not?" type, do appear naturally as for example in the investigations by Doebner and Goldin of representations of current algebras [20, 21]. The appearance of non-linear quantum mechanics in something as mathematically solid as unitary representations of the diffeomorphism group suggests that they may be more than just speculative curiosities. Their presence in such mathematical contexts must be clarified, and their possible physical relevance considered.

The argument against a nonlinear theory of Doebner-Goldin type is essentially the following: 1) The theory is nonrelativistic, 2) as such, it must be a nonrelativistic limit of a *nonlinear* relativistic theory, *but* 3) it has been shown that nonlinear relativistic theories are impossible, *ergo*, 4) the Doebner-Goldin equations are physically irrelevant [22].

Though the argument seems fairly convincing, there have appeared several proposals for circumventing the difficulties. One is by Doebner, Goldin and Natterman [23] in relation to the Doebner-Goldin theories themselves. A more careful analysis of the measurement process, relating all measurements to position measurements, shows that non-linearity in of itself does not lead to conflicts with relativity as certain non-linear equation are now equivalent to linear ones through a non-linear gauge transformation. This in itself in not too surprising as it amounts to the introduction of curvilinear coordinates in Hilbert space, and so the usual linear processes *look* nonlinear. The specifics of the gauge transformation may however be physically significant. Another proposal is due to Czachor [24] who introduces density matrix evolution that is not reducible to evolution of its component mixtures (the non-uniqueness of which is behind the causality problems). This can only work if somehow density matrices have a more fundamental significance than what is usually attributed to them, making the theory more radical then what may seem at first sight. In any case these two approaches are not manifestly covariant so to what extent conflicts with relativity can be overcome is not at all clear.

A careful analysis of what really goes into the difficulties of reconciling nonlinearity with relativity reveals the striking role played by the projection hypothesis or some modification thereof. This hypothesis in turn is based on the notion of self-subsistent physical state which obviously is a frame related notion. It is this frame-dependence that cannot be reconciled with relativity in the alternative theories. Abandoning such a frame-dependent notion would mitigate arguments against alternatives and opens up a true possibility of modifying quantum mechanics while maintaining *manifest* Lorentz covariance. Indeed a nonlinear version of the consistent histories approach seems to be compatible with relativity.

3. CONSISTENT HISTORIES, LINEAR AND NON-LINEAR

Let us review briefly the notions involved in the consistent histories approach [2, 3]. Let Ψ be a normalized state vector in Hilbert space, and for each $i = 1, \ldots, n$ let $P_j^{(i)}$ where

$j = 1, 2, \ldots, n_i$, be a finite resolution of the identity. A *history* is state vector of the form $P_\alpha \Psi = P^{(n)}_{\alpha_n} \cdots P^{(j)}_{\alpha_j} \cdots P^{(2)}_{\alpha_2} P^{(1)}_{\alpha_1} \Psi$. Let $p_\alpha = ||P_\alpha \Psi||^2$. One interpretation of the above quantities is that Ψ is a Heisenberg state and that $P^{(i)}_j$ is the spectral resolution of a Heisenberg observable $A^{(i)} = \sum \lambda^{(i)}_j P^{(i)}_j$ at time t_i where $t_1 < t_2 < \cdots < t_{n-1} < t_n$. In this case p_α is the joint probability of getting the sequence of outcomes $\lambda^{(1)}_{\alpha_1}, \ldots, \lambda^{(n)}_{\alpha_n}$ in a sequence of measurements that correspond to the observables $A^{(1)}, \ldots, A^{(n)}$. The coherent histories interpretation of quantum mechanics however goes beyond this viewpoint and in certain special cases interprets p_α as the probability of the history Q_α even if no actual measurements are made. It is a way of assigning probabilities to alternate views of the quantum state Ψ, corresponding to the possible different sequences $\alpha = (\alpha_1, \ldots, \alpha_n)$. Such an attitude is maintained only if a condition, called *consistency*, is satisfied by the set of alternative histories. This condition is the requirement that the quantum probabilities p_α behave as classical probabilities under coarsening of the histories, where by coarsening we mean replacing the resolutions of the identity by coarser ones by summing some of the projectors.

The most naive way to adapt the consistent histories approach to non-linear quantum mechanics is to replace the linear projectors $P^{(i)}_j$ by non-linear operators $B^{(i)}_j$ and so introduce *non-linear histories* $B_\alpha \Psi$, with the corresponding probability function $b_\alpha = ||B_\alpha \Psi||^2$. Such expressions are in fact the correct ones for a succession of measurements for the Goldin-Doebner-Natterman theories.

To complete the program and have an *interpretation* of this non-linear quantum mechanics, similar to the consistent histories approach of linear quantum mechanics, one needs to address the notions of coarsening and consistency. We shall not pursue this here. One wants Lorentz covariance, and the most naive way to get it is to assume that there is a unitary representation $U(g)$ of the Poincaré group along with an action ϕ_g of the same on suitable non-linear operators such that it makes sense to talk of the transformed histories $\tilde{B}_\alpha \tilde{\Psi} = \tilde{B}^{(n)}_{\alpha_n} \cdots \tilde{B}^{(j)}_{\alpha_j} \cdots \tilde{B}^{(2)}_{\alpha_2} \tilde{B}^{(1)}_{\alpha_1} \tilde{\Psi}$, where $\tilde{B} = \phi_g(B)$ and $\tilde{\Psi} = U(g)\Psi$. Lorentz covariance would then be expressed through the statement $\tilde{b}_\alpha = b_\alpha$. Such a scheme holds in the Goldin-Doebner-Natterman case for Euclidean and Galileian covariance.

Now it should not be very hard to implement the above scheme without further constraints, but for an interesting theory one should require a locality condition that would preclude superluminal signals. It would only be then that one could say that one has overcome the relativistic objections to non-linear theories.

4. FREE QUANTUM FIELD MODELS

Consider a free neutral scalar relativistic quantum field. For each limited space-time region $\mathcal{O}$ let $\mathcal{A}(\mathcal{O})$ be the algebra of observables associated to $\mathcal{O}$. Consider now a set of limited space-time regions $\mathcal{O}_1, \ldots \mathcal{O}_n$ which are so disposed that for any two, either all points of one are space-like in relation to all points of the other, or they are time-like. Assume the regions are numbered so that whenever one is in the time-like future of another, then the first one has a greater index. Let $P_i \in \mathcal{A}(\mathcal{O}_i)$ be an orthogonal projections. Let Ψ represent a heisenberg state in some reference frame and prior to all measurements. The probability to obtain all the outcomes represented by the projections is $||P_n \cdots P_2 P_1 \Psi||^2$. A nonlinear version can now be formulated by an *ad-hoc* replacement of P_i by B_i, a possibly non-linear operator, likewise somehow associated to the region $\mathcal{O}_i$, whenever there is a region $\mathcal{O}_j$ that is time-like past to the given one. This effectively differentiates between space-like and time-like conditional probabilities. For this to be consistent, relativistic, and causal, the operators B_i have to satisfy certain constraints [1], which are fairly straightforward to formulate. A consequence of these constraints is absence of superluminal signals, so that if the constraints can be met, we would have an explicit example of a non-linear relativistic quantum mechanics.

A very simple way of satisfying the constraints is to set in the time-like case $B_j = BP_jB^{-1}$ where B is an invertible not-necessarily linear operator that is Poincaré invariant, $U(g)BU(g)^* = B$. In this case the only surviving constraint is $[BPB^{-1}, Q] = 0$ if P and Q are orthogonal projectors belonging to space-like separated regions. A stronger condition would be $[B\phi(x)B^{-1}, \phi(y)] = 0$ for x and y space-like separated, where $\phi(x)$ is the free quantum field.

Now it is not hard to find Poincaré invariant non-linear operators, the difficulty is satisfying the above constraint, and the presence of B^{-1} is a major hindrance. Considering B to be close to the identity $B = I + K$, then to first order in K the constraint becomes $[[K, \phi(x)], \phi(y)] = 0$ for x and y space-like separated. A detailed study of this constraint is still in progress, but there seems to be no real obstruction to satisfying it, so that it seems that, at least formally, a causal non-linear relativistic quantum mechanics is possible.

Schemes based on non-linear histories which differentiate between space-like and time-like joint-probabilities in principle should exhibit physical effects as one crosses the light cone. Thus in a typical photon EPR-type correlation experiment, one can delay the light-ray on one side so that at a certain point the detector events become time-like. In crossing the light cone, an effect should be present that was not foreseen by the linear theory. This happens in the models above as $||PQ\Phi||^2$ suddenly becomes $||BQ\Phi||^2$. Call such theories *future light-cone singular*. On the other hand we have brought forth plausibility arguments that theories that do not suffer such discontinuities at the future light-cone are necessarily linear, so a true verification of non-linearity would involve light-cone experiments. Note that in such theories, the notion of self-subsisting physical state has to be abandoned, since otherwise by deterministic evolution of the state to a time before the first measurement, we can consider it as being created in a different space-time region which would have a different spatio-temporal relationship with the second measurement and so lose the specific light-cone relation needed for future light-cone singular theories. Such theories provide patterns of joint probabilities not analyzable in terms of autonomous interpolating entities.

5. CONCLUSIONS

The first conclusion that one can draw is that there is a close relationship between space-time structure and quantum mechanics. Space-time imposes universally valid constraints on physical theories and the universality of quantum mechanics starts to become less mysterious. However, this very universality now poses a self-consistency question. The general analysis pertained to *any* measurements, including those that reveal the structure of space-time itself. Thus by consistency, space-time itself must exhibit quantum phenomena. But the starting point of the analysis was classical space-time, so to be completely consistent we must redo the whole argument starting with quantum space-time and arrive at constraints that are consistent with the quantum structure of space-time at the start of the analysis.

Such self-consistency restricts the possibilities of physical theories yet further. Thus though non-linear relativistic quantum mechanics seems to be logically consistent if we abandon the notion of self-subsisting physical states and adopt a consistent histories approach, such theories are likely to be future light-cone singular. This characteristic depends on a sharp distinction between space-like and time-like, but if space-time itself has to be quantized to conform to the universal constraints, the distinction becomes blurred. At first sight, the consequence of this blurring would be to diminish the light-cone effect, and we see a renormalization group in action. For consistency one has to be at a fixed point, which would be linear quantum mechanics. The final conjecture is then that the linearity of quantum mechanics is a renormalization effect of quantum gravity. Though the fluctuations of the light cone has been up to now considered a conceptual problem with quantized space-time, it may be that it is a fundamental ingredient in explaining why quantum mechanics is the way it is.

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